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A Study of Fuzzy σ-Baire, Supra-Baire, and Soft Baire spaces

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Foreword

It is with great pleasure that I introduce “A Study of Fuzzy σ-Baire, Supra-Baire, and Soft Baire Spaces.” This volume assembles a substantial body of original research that pushes the frontiers of modern topology into the realm of fuzzy and soft mathematics. Curated and edited by Dr. E. Poongothai, Dr. P. Balaganesan, and myself, this work is a specialized collection of research articles previously published across several distinguished platforms. These include the *International Journal of Fuzzy Mathematics and Systems*, *Annals of Fuzzy Mathematics and Informatics*, *Advances in Fuzzy Mathematics*, the *Journal of Physics: Conference Series* (IOP Publishing), and *Advances in Mathematics: Scientific Journal*, among others. By consolidating these disparate studies into a single thematic volume, the editors have produced a work that will become a cornerstone reference for anyone working with uncertainty-laden structures.

Why this book matters

Classical topology, rooted in crisp point-set concepts, has for decades supplied the language for continuity, convergence, and dimension. Over the past sixty years, however, the need to model imprecision has driven mathematicians to blend traditional topological ideas with revolutionary frameworks.

This journey began with Lotfi A. Zadeh’s seminal introduction of Fuzzy Set Theory, which allowed for degrees of membership rather than binary belonging. My own extension of this logic into Neutrosophic Sets—which accounts for degrees of truth, indeterminacy, and falsity—further expanded this landscape, emphasizing that the "neutral" or "indeterminate" zone is often as critical as the membership itself. Subsequently, the introduction of Soft Set Theory by Molodtsov provided a parameterised tool for dealing with uncertainty. The resulting fuzzy topologies, supra-topologies, and soft topologies retain the elegance of their classical ancestors while admitting these nuanced layers of membership and parameterised indeterminacy.

Within this broad landscape, Baire spaces occupy a privileged position. Their defining property—that a countable union of nowhere-dense sets cannot exhaust the whole space—underpins many deep results in analysis. Extending this notion to fuzzy and soft contexts is far from trivial; the interplay between interior, closure, and degree of membership demands new techniques and fresh insight.

Thematic Pillars

The present volume meets this challenge through three core areas of focus:

- **Fuzzy σ-Baire Spaces:** A treatment of σ-Baire properties within fuzzy Fσ-sets, exploring characterisations and connections with fuzzy P-spaces.
- **Supra-Baire Spaces:** Investigations into Baire-type behavior in higher-order structures, including results on supra-hyperconnectedness and supra-submaximality.
- **Soft Baire Spaces:** Incorporating soft parameters to capture layers of uncertainty, with applications to decision-making and information systems.

The contributors

This book is the result of close collaboration among a dedicated group of scholars: Prof. Ganesan Thangaraj (Thiruvalluvar University), Dr. Eswaran Poongothai, Dr. S. Divyapriya, Dr. A. Celine, Dr. G. Priya, and M. Nalini (Shanmuga Industries Arts & Science College). Their combined expertise ensures that each chapter balances rigorous proof-theoretic development with insightful examples.

Concluding remarks

The study of Baire phenomena in fuzzy, supra, and soft environments is still evolving, and this volume marks a decisive step forward. I commend the editors and all authors for their scholarly precision and vision. Their work will undoubtedly inspire the next generation of mathematicians to continue expanding the horizons of fuzzy topology.

Dr. Florentin Smarandache

On Fuzzy σ-Baire Spaces

Ganesan Thangaraj, Eswaran Poongothai

Abstract

In this paper the concepts of fuzzyσ-Baire spaces are introduced and characterizations of fuzzy σ-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

KEYWORDS: Fuzzy F_σ-set, fuzzyG_δ-set, fuzzy nowhere dense set, Fuzzyσ-nowhere denses et, Fuzzy σ-first category, Fuzzy σ-second category and Fuzzyσ-Baire spaces,

2000 AMS Classification: 54A40, 03E72.

INTRODUCTION.

The fuzzy concept has invaded almostall branchesofMathematicsever since the introduction offuzzy setby L. A. ZADEH [12] . Thetheoryof fuzzy topologicalspaces was introducedand developed by C. L. CHANG [3] . Since thenmuchattention has been paid togeneralize thebasicconcepts of GeneralTopologyinfuzzy settingand thus a modern theoryof fuzzy topology hasbeen developed. Inrecentyears, fuzzy topology has been foundto be very useful insolvingmanypractical problems. In [5] El. Naschieshowed that thenotion offuzzy topology mightbe relevant to Quantum Particle Physics in connection with String Theory. It has been shown that the fuzzyKähler manifoldswhich are based on a topology, play the important rule in $\in^{\infty}$theory. In this paper we introduce the concepts of fuzzy σ-Baire spaces. Alsowe discussseveralcharacterizationsof fuzzyσ-Bairespaces.

Several examples are given to illustrate theconcepts introduced inthis paper.

2. PRELIMINARIES

Byafuzzy topologicalspace we shall meana non –emptyset Xtogetherwitha fuzzy topologyT (in the sense of Chang) and denote it by (X, T).

DEFINTION 2. 1: Let λandμ beany twofuzzy sets in (X, T). Then wedefine$\lambda \vee \mu$: X $\rightarrow$ [0, 1] as follows: $(\lambda \vee \mu)(x) = \text{Max}\{\lambda(x), \mu(x)\}$. Alsowe define $\lambda \wedge \mu$: X $\rightarrow$ [0, 1] as follows: $(\lambda \wedge \mu)(x) = \text{Min}\{\lambda(x), \mu(x)\}$.

DEFINTION 2. 2`` [1] . : Let (X, T) be any fuzzy topological space and λbe any fuzzy setin (X, T). We define $Cl(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1-\mu \in T\}$and $int(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$.

For anyfuzzy setina fuzzy topological space(X, T), itiseasy to see that 1— cl (λ)=int (1 — λ)and 1 – int(λ)=cl (1 — λ)

DEFINTION 2. 3: Let (X, T) and (Y, S) be any two fuzzy topological spaces. Letfbe a function from the fuzzy topological space (X, T) tothe fuzzy topological space (Y, S).

Let λbea fuzzy set in (Y, S). The inverse imageof λ underf written as $f^{-1}(\lambda)$ is the fuzzy set in(X, T) defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in X$. Also the image of λin (X, T) under f written as f (λ) is the fuzzy set in(Y, S)defined by

$$f(\lambda)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \lambda(x) & \text{if } f^{-1}(y) \text{ is non} - \text{empty}; \\ 0 & \text{otherwise.} \end{cases}$$

for each $y \in Y$.

Lemma2. 1 [3] : Letf: (X, T) $\rightarrow$(Y, S) be amapping. Forfuzzy setsλ andμof (X, T) and (Y, S) respectively, the following statementshold:

1. $ff^{-1}(\mu) \leq \mu$;
2. $f^{-1}f(\lambda) \geq \lambda$;
3. $f(1-\lambda) \geq 1 - f(\lambda)$;
4. $f^{-1}(1-\mu) = 1 - f^{-1}(\mu)$;
5. Iffis one-to-one, then $f^{-1}f(\lambda) = \lambda$;
6. If f is onto, then $ff^{-1}(\mu) = \mu$;
7. If f isone-to-one andonto, then $f(1-\lambda) = 1 - f(\lambda)$.

DEFINTION 2. 4 [8] : A fuzzyset λ in a fuzzytopological space (X, T) is calledfuzzy dense if there existsno fuzzy closedsetμ in (X, T) such that$\lambda < \mu < 1$.

DEFINITION 2. 5 [3] : A fuzzy set λ in a fuzzy topologicalspace (X, T) is called a fuzzy F_σ-set in(X, T) if $\boldsymbol{\lambda} = \bigvee_{i=1}^{\infty} \boldsymbol{\lambda_i}$, where$1 - \lambda_i \in T$ for $i \in I$.

DEFINITION 2. 6 [3] : A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzyG_δ-set in (X, T) if$\boldsymbol{\lambda} = \bigwedge_{i=1}^{\infty} \boldsymbol{\lambda_i}$, where$\lambda_i \in T$ for $i \in I$.

DEFINITION 2. 7 [1] : Afuzzyset λinafuzzytopologicalspace(X, T) is called a fuzzy semi-open if $\lambda \leq$ clint(λ). The complementof λ in (X, T)iscalledafuzzy semi-closed set in (X, T).

DEFINITION 2. 8 [10] : A fuzzytopologicalspace (X, T)is called an fuzzy open hereditarilyirresolvable space ifint cl (λ)≠0, then int(λ)≠ 0 for any non-zero fuzzy set in (X, T) .

DEFINITION 2. 9. [8] : A fuzzy topological space (X, T) is called fuzzy first category if the fuzzy set $\mathbf{1_X}$ is a fuzzy first category set in (X, T). That is, $1_X = \bigvee_{i=1}^{\infty}(\lambda_i)$where λ_i 's are fuzzynowhere dense sets in (X, T). Otherwise, (X, T) will be called a fuzzy second category space.

3. FUZZY σ-NOWHERE DENSESETS

Motivated by the classical concept introduced in [5] we shall now define:

DEFINITION 3. 1: Let(X, T) beafuzzy topological space. A fuzzy set λin (X, T) is called a fuzzyσ-nowhere dense setifλ is a fuzzy F_σ-set in (X, T) suchthat int (λ) = 0.

EXAMPLE 3. 1: Let X ={a, b, c}. The fuzzy sets λ, μ and υare definedon X as follows:

- λ: X → [0, 1] isdefined asλ (a) =0. 3; λ(b)=0. 7; λ (c)=0. 4.
- μ: X → [0, 1] isdefined as μ (a)=0. 5; μ (b) = 0. 4;μ (c) =0. 8.
- υ: X → [0, 1] isdefinedasυ(a) =0. 2; υ(b) =0. 4; υ (c) =0. 6.

Then, T = {0, λ, μ, υ, λ ∨ μ, λ ∨υ, λ ∧ μ, λ ∧υ, υ∨ (λ ∧ μ), 1} is clearly a fuzzy topology on X. Now consider the fuzzy setδ= [1 – (λ ∨ μ)] ∨ [1 – (λ ∨υ)] in (X, T). Then δ is a fuzzy F_σ-set in (X, T)and int (λ)=0 and henceδis a fuzzy σ-nowhere dense set in (X, T). The fuzzy set β = (1 –λ)∨(1 –μ) ∨(1 –υ)is a fuzzy F_σ-set in (X, T) and int (λ)≠ 0 and hence β isnot a fuzzy σ-nowhere dense set in (X, T).

REMARKS 3. 1: If λ and μare fuzzyσ-nowhere dense sets, then λ ∨ μ need not be a fuzzyσ-nowhere dense set in (X, T). For, consider the following example:

EXAMPLE 3. 2: Let X = {a, b, c}. The fuzzy sets λ, μ andυare definedon X as follows:

- λ: X → [0, 1] defined as λ (a) = 0. 6 ; λ(b)= 0. 7 ; λ (c) = 0. 5.
- μ: X → [0, 1] defined as μ (a) = 0. 5 ; μ (b) = 0. 4 ; μ (c) =0. 8.
- υ: X → [0, 1] defined as υ (a) = 0. 7 ; υ (b) = 0. 6 ; υ (c) =0. 5 .

Now α = [(1 – μ)∨{1 – (λ ∨ μ)}∨ {1 – (μ∨υ) }] and β = [(1 –λ)∨(1 –υ)] are fuzzy F_σ-sets in (X, T). Also int (α) = 0 and int (β) = 0. Therefore α and β are fuzzy σ-nowhere dense sets in (X, T). But α∨ βisnot afuzzyσ – nowhere dense set in (X, T),

since$\alpha \vee \beta$ isa fuzzy F_σ-set in (X, T)and int $(\alpha \vee \beta) = \mu \wedge \upsilon \neq 0$.

PROPOSITION 3. 1: In a fuzzy topological space (X, T)a fuzzy set λ isfuzzy σ-nowhere dense in (X, T) if and only if $1 - \lambda$ is a fuzzy dense and fuzzy G_δ-set in (X, T).

PROOF: Let λ bea fuzzy σ-nowhere dense set in (X, T). Then $\boldsymbol{\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)}$where $1 - \lambda_i \in T$, for $i \in I$ and int $(\lambda) = 0$. Then $1 - \text{int}(\lambda) = 1 - 0 = 1$ implies thatcl $(1 - \lambda) = 1$.

Also$(1 - \lambda) = 1 - \bigvee_{i=1}^{\infty}(\boldsymbol{\lambda_i}) = \bigwedge_{i=1}^{\infty}(1 - \lambda_i)$where$1 - \lambda_i \in T$, for $i \in I$. Hence we have $1 - \lambda$ is a fuzzy dense and fuzzy G_δ-set in (X, T).

Conversely, let λ be a fuzzy dense and fuzzy G_δ-set in (X, T). Then $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$where$\lambda_i \in T$, for $i \in I$. Now $(1 - \lambda) = 1 - \bigwedge_{i=1}^{\infty}(\lambda_i) = \bigvee_{i=1}^{\infty}(1 - \boldsymbol{\lambda_i})$.

Hence $1 - \lambda$ is a F_σ-set in (X, T) and int $(1 - \lambda) = 1 - \text{cl}(\lambda) = 1 - 1 = 0$. [since λ is a fuzzy dense] . Therefore $1 - \lambda$ is a fuzzy σ-nowhere dense set in (X, T).

PROPOSITION 3. 2: If λ is a fuzzy dense set in (X, T) such that $\mu \leq (1 - \lambda)$, where μ is a fuzzy F_σ-set in (X, T), then μ is a fuzzyσ-nowhere dense set in (X, T).

PROOF: Let λ be a fuzzy dense set in (X, T) such that $\mu \leq (1 - \lambda)$. Now $\mu \leq (1 - \lambda)$ implies that $\text{int}(\mu) \leq \text{int}(1 - \lambda) = 1 - \text{cl}(\lambda) = 1 - 1 = 0$ and hence $\text{int}(\mu) = 0$.

Therefore μ is a fuzzy σ-nowhere dense set in (X, T).

PROPOSITION 3. 3: If λ is a fuzzy F_σ-set and fuzzy nowhere dense set in (X, T), then$\lambda$is a fuzzyσ-nowhere dense set in (X, T).

PROOF: Now $\lambda \leq \text{cl}(\lambda)$ for any fuzzy set in (X, T). Then, $\text{int}(\lambda) \leq \text{intcl}(\lambda)$. Since λ is a fuzzy nowhere dense set in (X, T), $\text{intcl}(\lambda) = 0$ and hence $\text{int}(\lambda) = 0$ andλ is a fuzzy F_σ-set implies that $\lambda$is a fuzzy σ-nowhere dense set in (X, T).

REMARKS 3. 2: If λ is a fuzzy F_σ-set and fuzzy σ-nowhere dense set in (X, T), thenλ need not be a fuzzynowhere dense set in (X, T). For, consider the following example:

EXAMPLE 3. 3: Let X = {a, b, c}. The fuzzy sets λ, μ and$\upsilon$are definedon X as follows:

- λ: X $\rightarrow$ [0, 1] defined as λ (a) = 0. 8 ; λ(b)= 0. 6 ; λ (c) = 0. 7.
- μ: X $\rightarrow$ [0, 1] defined as μ (a) = 0. 6 ; μ (b) = 0. 9 ; μ (c) =0. 8.
- υ: X $\rightarrow$ [0, 1] defined as υ (a) = 0. 7 ; υ (b) = 0. 5 ; υ (c) =0. 9 .

Then T ={0, λ, μ, υ, $\lambda \vee \mu$, $\lambda \vee \upsilon$, $\mu \vee \upsilon$, $\lambda \wedge \mu$, $\lambda \wedge \upsilon$, $\mu \wedge \upsilon$, $\lambda \vee (\mu \wedge \upsilon)$, $\mu \vee (\lambda \wedge \upsilon)$, $\upsilon \vee (\lambda \wedge \mu)$, 1} is a fuzzy topology on X. Now the fuzzy set $\eta = (1 - \mu) \vee (1 - [\lambda \wedge \upsilon]) \vee (1 - \upsilon)$ and int $(\eta) = 0$ and hence η is a fuzzy F_σ-set and fuzzy σ-nowhere dense set in (X, T) but η is not a fuzzynowhere dense set in (X, T) since int cl$(\eta) \neq 0$.

PROPOSITION 3. 4: If(X, T) is a fuzzy open hereditarily irresolvable space, any fuzzy σ-nowhere dense set in (X, T) is a fuzzy nowhere dense set in (X, T).

PROOF: Let λ be a fuzzy σ-nowhere dense set in an fuzzy open hereditarily irresolvable space (X, T). Thenλ is a fuzzy F_σ-set in (X, T) suchthat int (λ) =0. Since(X, T) is anfuzzy open hereditarily irresolvable space, int(λ)= 0 implies that int cl(λ) =0. Henceλ is a fuzzy nowhere dense set in (X, T).

DEFINITION 3. 2: Let(X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called fuzzy σ-first category if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i' s\ are$ fuzzyσ-nowheredense sets in (X, T). Any otherfuzzy set in (X, T) is said to be fuzzy σ-second categoryin (X, T) .

DEFINITION 3. 3: Let λ be a fuzzy σ-first category set in (X, T). Then 1 – λ is called a fuzzy σ-residual set in (X, T).

DEFINITION 3. 4: A fuzzy topological space (X, T) is called fuzzy σ-first category if the fuzzy set $\mathbf{1_X}$ is a fuzzy σ-first category set in (X, T). That is, $1_X = \bigvee_{i=1}^{\infty}(\lambda_i)$where λ_i 's are fuzzyσ-nowhere dense sets in (X, T). Otherwise, (X, T) will be called a fuzzy σ-second category space.

4. FUZZY σ-BAIRE SPACE

Motivated by the classical concept introduced in [6] we shall now define:

DEFINITION 4. 1: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy σ-Baire Spaceif int $(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$ where $\lambda_i' s\ are$ fuzzy σ-nowheredensesets in (X, T).

EXAMPLE 4. 1: Let X = {a, b, c}. The fuzzy sets λ, μ andυ are defined on X as follows:

- λ: X → [0, 1] is defined as λ (a)= 0. 8; λ(b) = 0. 6 ; λ (c) = 0. 7.
- μ: X → [0, 1] isdefined as μ (a) = 0. 6 ; μ (b) =0. 9; μ (c) = 0. 8.
- υ: X → [0, 1] is defined as υ (a) = 0. 7 ; υ (b) = 0. 5 ; υ (c) =0. 9.

Then T = {0, λ, μ, υ, λ ∨ μ, λ ∨υ, μ ∨υ, λ ∧ μ, λ ∧υ, μ ∧υ, λ∧(μ ∨υ), λ∨ (μ ∧υ), μ∧ (λ ∨υ), μ∨(λ ∧υ), υ∧ (λ ∨ μ), υ∨ (λ ∧ μ), 1} is a fuzzy topology on X. Now

- α= (1 –λ)∨(1 – [λ∨ (μ ∧υ)]) ∨ [1 – (λ ∨ μ∨υ)] and int (α) = 0.
- β = (1 –μ) ∨(1 – [λ∨υ]) ∨ [1 – (μ∨(λ ∧υ))] and int (β) = 0.
- = (1 –υ) ∨(1 – [λ∨μ]) ∨ [1 –(λ ∧μ)] ∨(1 – [μ ∨υ]) and int (δ) = 0
- η= (1 – [λ ∧υ]) ∨(1 – [μ ∧υ]) ∨ [1 –(λ ∧ [μ ∨υ] , ∨(1 – [μ∧(λ ∨υ)]) and int (η) = 0.

Thenα, β, δ and $\eta$are fuzzy σ-nowhere dense sets in (X, T) and also int $(\alpha \vee \beta \vee \eta \vee \eta) = 0$ and therefore (X, T) is a fuzzy σ-Baire Space.

PROPOSITION4. 1: Let (X, T) be a fuzzy topological space. Then the following are equivalent:

1. (X, T) is a fuzzy σ-Baire space.
2. Int (λ) = 0 for every fuzzy σ-first category set λ in (X, T).
3. cl (μ) = 1 for every fuzzy σ-residual set μ in (X, T).

PROOF: (1) $\Rightarrow$(2). Let λ be a fuzzy σ-first category set in (X, T). Then $\lambda = (\bigvee_{i=1}^{\infty}(\lambda_i))$ where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Then, we have int (λ)=int$(\bigvee_{i=1}^{\infty}(\lambda_i))$.

Since (X, T) is a fuzzy σ-Baire space, int$(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$. Hence int(λ)= 0 for any fuzzy σ-first category set λ in (X, T).

(2) $\Rightarrow$ (3). Let μ be a fuzzy σ-residual set in (X, T). Then $(1 - \mu)$ is a fuzzy σ – first category set in (X, T). By hypothesis, int $(1 - \mu) = 0$. Then $1 -$ cl $(\lambda) = 0$. Hence cl $(\lambda) = 1$for any fuzzy σ-residual set μ in (X, T).

(3) $\Rightarrow$(1). Let λ be a fuzzy σ-first category set in (X, T). Then $\lambda = (\bigvee_{i=1}^{\infty}(\lambda_i))$ where λ_i's are fuzzy σ-nowhere dense sets in(X, T). Now λ is afuzzy first σ-category set in (X, T) implies that $(1 - \lambda)$ is a fuzzyσ-residual set in(X, T). By hypothesis, we have cl $(1 - \lambda) = 1$. Then $1 -$ int $(\lambda) = 1$. Henceint $(\lambda) = 0$. Thatis, int$(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy σ – nowheredense sets in (X, T). Hence (X, T) is a fuzzy σ-Bairespace.

PROPOSITION4. 2: If $cl(\bigwedge_{i=1}^{\infty}(\lambda i)) = 1$ where λ_i's arefuzzy dense G_δ-sets in (X, T), then (X, T) is a fuzzy σ-Baire space.

PROOF: Now $cl(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 1$ implies that $1 - cl(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 0$. Then we have int $(1 - (\bigwedge_{i=1}^{\infty}(\lambda_i)) = 0$, which implies that int $(\bigvee_{i=1}^{\infty} (1 - \lambda_i) = 0$. Let$\mu_i = 1 - \lambda_i$.

Then int$(\bigvee_{i=1}^{\infty}(\mu_i)) = 0$. Since $\lambda_i$is a fuzzy dense G_δ-setin(X, T), by proposition 3. 1, $1 - \lambda_i$ isa fuzzy σ-nowhere dense set in(X, T). Henceint$(\bigvee_{i=1}^{\infty}(\mu_i)) = 0$, where $\mu_i's$ are fuzzy σ-nowhere dense setsin(X, T). Therefore(X, T) is a fuzzy σ-Bairespace.

PROPOSITION 4. 3: If the fuzzy topological space (X, T) is a fuzzyσ-Baire space, then (X, T) is a fuzzyσ-second category space.

PROOF: Let (X, T) be a fuzzyσ-Baire space. Then int$(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Then$\bigvee_{i=1}^{\infty}(\lambda_i) \neq 1_x$ [Other wise, $\bigvee_{i=1}^{\infty}(\lambda_i) = 1_x$ implies thatint $(\bigvee_{i=1}^{\infty}(\lambda_i)) =$ int $1_x = 1_x$, which implies that 0 = 1, a contradiction] . Hence (X, T) is a fuzzyσ-second category space.

REMARKS: The converse of the above proposition need not be true. A fuzzy σ-second category space need not be a fuzzy σ-Baire space. For, consider the following example:

EXAMPLE4. 2: Let X = {a, b, c} and λ, μ, υ be the fuzzy sets defined on X as follows:

- λ: X → [0, 1] isdefined as λ (a) = 0. 5; λ(b) = 0. 6 ; λ (c)= 0. 7.
- μ: X → [0, 1] isdefined as μ (a) = 0. 8 ; μ (b) = 0. 4; μ (c) =0. 5.
- υ: X → [0, 1] isdefined as υ(a) = 0. 7 ; υ(b) = 0. 5 ; υ (c) =0. 8

Then T = { 0, λ, μ, υ, λ ∨ μ, λ∨υ, μ∨υ, λ∧μ, λ ∧υ, μ∧υ, λ ∨(μ∧υ), μ∨ (λ ∧υ), υ∧ (λ ∨ μ), λ ∨ μ∨υ, 1 }is a fuzzy topology on X. Now the fuzzy set α = { (1 – μ) ∨ (1-υ)∨ (1 – (λ∨ μ))∨(1 – (λ∨υ))∨ (1 – (μ∨υ))∨(1 – (λ∨μ∨υ)}, is a fuzzy F_{σ}-set in (X, T) and int (α) = 0 and hence is a fuzzy σ-nowhere dense set in (X, T).

Also β={ (1 –λ) ∨(1 – [λ ∧υ]) ∨(1 – (λ∨ [μ∧υ]))∨((1 – [μ∨ (λ ∧υ)])∨(1 – [υ∧ (λ∨μ)])} Is a fuzzy F_{σ}-set in (X, T) and int (β) = 0 and hence is a fuzzy σ-nowhere dense set in (X, T). Now (α∨β) ≠ 1_x . Therefore (X, T) is a fuzzyσ-second category space. But int (α∨β) =λ∧μ≠0 and therefore (X, T) is not a fuzzyσ-Baire space.

REMARKS: A fuzzy σ-Baire spaceneed not be a fuzzyBairespace. For, consider the following example:

EXAMPLE 4. 3: Let X = {a, b, c}. The fuzzy sets λ, μ andυaredefinedon X as follows:

- λ: X → [0, 1] is defined as λ(a) = 1; λ(b)= 0. 2 ; λ (c) = 0. 7.
- μ: X → [0, 1] is defined as μ(a) = 0. 3 ; μ (b) = 1; μ (c) =0. 2.
- υ: X → [0, 1] is defined as υ (a) = 0. 7 ; υ(b) = 0. 4 ; υ(c)=1.

Then T = { 0, λ, μ, υ, λ ∨ μ, λ∨υ, μ∨υ, λ∧μ, λ ∧υ, μ∧υ, μ∨ (λ ∧υ), υ∨ (λ∧μ), υ∧ (λ ∨ μ), λ ∨ μ∨υ, 1 }is a fuzzy topology on X. Now 1 – λ, 1 – μ, 1 – υ, 1 – (λ ∨ μ), 1 – (λ∨υ), 1– (μ∨υ), 1– (λ ∧υ), 1– [μ∨ (λ ∧υ)] , 1– [υ∨ (λ∧μ)] , 1– [υ∧ (λ ∨ μ)] arefuzzy nowhere dense sets in (X, T). 1 – (λ∧μ) = {(1 – λ) ∨ (1 – μ) ∨ (1-υ)∨ (1 – (λ∨ μ))∨(1 – (λ∨υ))∨ (1 – (μ∨υ))∨(1– [μ∨ (λ ∧υ)])∨ (1– [υ∨ (λ∧μ)])∨ (1– [υ∧(λ ∨ μ)])} . Therefore, 1 – (λ∧μ) is a fuzzy first categoryset in(X, T). But int (1 – (λ∧μ)) = λ∧μ ≠0. Hence (X, T) is not a fuzzy Baire Space. Now α= (1 –υ) ∨(1 – [λ∨υ]) ∨ [1 – (μ∨υ)] and int (α) = 0. β = (1 –λ) ∨(1 – [λ∨μ]) ∨ [1 – (λ ∧υ)] ∨(1– [μ∨ (λ ∧υ)] ∨ [1– [υ∨ (λ∧μ)] and int (β) = 0. Hence α and β arefuzzy σ-nowhere dense sets in (X, T). Now the fuzzy (α∨ β)is a fuzzyσ-first categoryset in (X, T)and int ((α∨ β)=0. Hence (X, T) is a fuzzy σ-Baire space.

PROPOSITION 4. 4: If the fuzzy topologicalspace (X, T) is a fuzzyσ-Baire and fuzzy open hereditarilyirresolvabespace, then (X, T) is a fuzzyBaire Space.

PROOF: Let (X, T)be a fuzzyσ-BaireSpace and fuzzyopen hereditarily irresolvabe space. Then, $\text{int}(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$where λ_i's are fuzzy σ-nowhere dense sets in (X, T).

By proposition 3. 4, λ_i' s are fuzzynowhere dense sets in (X, T). Hence, $\text{int}(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i' s are fuzzy nowhere dense sets in (X, T). Therefore (X, T) is a fuzzyBaire Space.

PROPOSITION 4. 5: Ifthe fuzzy topologicalspace (X, T) is a fuzzy BaireSpace and if the fuzzy nowhere dense sets in (X, T) are fuzzy F_σ-sets in (X, T), then (X, T) is a fuzzyσ-Baire Space.

PROOF: Let (X, T)be a fuzzyBaireSpacesuch that every fuzzy nowhere dense set $\lambda_i$is a fuzzy F_σ-set in (X, T). Then, $\text{int}(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy nowhere dense sets in (X, T). By proposition 3. 3, $\lambda_i$is a fuzzyσ-nowhere dense set in (X, T). Henceint$(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy σ-nowhere dense set in (X, T). Therefore (X, T) is a fuzzyσ-Baire Space.

PROPOSITION 4. 6: Let (X, T) be a fuzzy topologicalspace. If$\bigwedge_{i=1}^{\infty}(\lambda i) \neq 0$, where λ_i's are fuzzy dense and fuzzy G_δ-sets in (X, T), then (X, T) is a fuzzy σ-second category space.

PROOF: Now$\bigwedge_{i=1}^{\infty}(\lambda_i) \neq 0$ implies that $1 - (\bigwedge_{i=1}^{\infty}(\lambda_i)) \neq 1 - 0 = 0$. Then we have $(\bigvee_{i=1}^{\infty}(1 - \lambda_i)) \neq 1$. Since $\lambda_i$is a fuzzy dense and fuzzy G_δ-setin(X, T), by proposition 3. 1, $1 - \lambda_i$ is a fuzzy σ-nowhere dense set in(X, T). Hence $\bigvee_{i=1}^{\infty}(1 - \lambda_i) \neq 1$, where $(1 - \lambda_i)$'s are fuzzy σ –nowheredense sets in(X, T). Hence (X, T) is not a fuzzy σ –first category space. Therefore (X, T) is a fuzzy σ-second category space.

5. FUNCTIONSAND FUZZYσ-BAIRE SPACES

DEFINITION 5. 1 [1] : A functionf: (X, T) → (Y, S) from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), is said to fuzzy open if the image of every fuzzy open set in (X, T), is fuzzy open in (Y, S).

Definition 5. 2 [9] : A function f: (X, T) → (Y, S) from a fuzzy topologicalspace (X, T) into another fuzzy topological space (Y, S), is called fuzzy contra-continuous if $f^{-1}(\lambda)$ is fuzzy closed (open) in (X, T), for each fuzzy open (closed) set λ in (Y, S).

Proposition 5. 1: If f: (X, T) → (Y, S) is an fuzzy contra –continuous and fuzzy open function froma topological space(X, T)onto a fuzzy open hereditarily irresolvable space(Y, S), then (Y, S) is a fuzzy σ-Baire space.

PROOF: Let λ be a fuzzy σ-first category set in (Y, S). Then $\lambda = (\bigvee_{i=1}^{\infty}(\lambda_i))$ where λ_i's are fuzzy σ-nowhere dense sets in (Y, S). Suppose that int (λ) ≠ 0. Then there exists an fuzzy open set $\mu \neq 0$ in (Y, S) such that $\mu \leq \lambda$. Then $f^{-1}(\mu) \leq f^{-1}(\lambda) = f^{-1}((\bigvee_{i=1}^{\infty}(\lambda_i)) = \bigvee_{i=1}^{\infty} f^{-1}(\lambda_i)$. Hence$f^{-1}(\mu) \leq \bigvee_{i=1}^{\infty} f^{-1}(\text{cl}(\lambda_i))$ Since f is afuzzy contra – continuousfunction and cl (λ_i) is a fuzzy closed set in (Y, S), $f^{-1}(\text{cl}(\lambda_i))$ is fuzzy open in(X, T). Hence we have, $f^{-1}(\mu) \leq \bigvee_{i=1}^{\infty} f^{-1}(\text{cl}(\lambda_i)) = \bigvee_{i=1}^{\infty} \text{int}(f^{-1}(\text{cl}(\lambda_i))$

Since f is fuzzy open and onto, int $(f^{-1}(\lambda_i)) \leq (f^{-1}$ int $(\lambda_i))$ and therefore we have $f^{-1}(\mu) \leq \bigvee_{i=1}^{\infty} f^{-1}(\text{int cl}(\lambda_I))$. Since (Y, S) is a fuzzy open hereditarily irresolvable space, by proposition 3. 4, the fuzzyσ-nowhere dense setsλ_i's are fuzzy nowhere dense sets in (Y, S) . Hence we have int cl $(\lambda_i)=0$. Then $f^{-1}(\mu) \leq \bigvee_{i=1}^{\infty} f^{-1}(0) = 0$.

That is, $f^{-1}(\mu) \leq 0$ and hence $f^{-1}(\mu) = 0$ which implies that $\mu = 0$, a contradiction to $\mu \neq 0$. Hence we must have int$(\lambda) = 0$ where λ is a fuzzy σ-first categorysetin (Y, S). Hence by proposition 4. 1, (Y, S) is a fuzzy σ-Baire space.

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Fuzzy σ-Baire spaces and functions

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Abstract. In this paper some results concerning functions that preserve fuzzy σ-Baire spaces in the context of images and preimages are obtained.

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Keywords: Fuzzy σ-dense, Fuzzy σ-nowhere dense, Fuzzy σ-Baire space, Fuzzy Volterra space, Fuzzy continuous, Somewhat Fuzzy continuous, Fuzzy open and Somewhat Fuzzy open functions.

1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper [8] in the year 1965. Thereafter the paper of C. L. Chang[3] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concept of σ-Baire spaces in fuzzy setting was introduced and studied by the authors in [5]. The purpose of this paper is to study the effect of fuzzy continuity, fuzzy openness and some decompositions of fuzzy continuity and fuzzy openness on fuzzy σ-Baire spaces.

2. Preliminaries

Now we introduce some basic notions and results that are used in the sequel. In this work, by a fuzzy topological space, we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X, T). The interior, closure and the complement of a fuzzy set λ will be denoted by $int(\lambda), cl(\lambda)$ and $1 - \lambda$ respectively.

Definition 2.1. Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1-\mu \in T\}$ and $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$.

For any fuzzy set λ in a fuzzy topological space (X,T), it is easy to see that $1 - cl(\lambda) = int(1-\lambda)$ and $1 - int(\lambda) = cl(1-\lambda)$[1].

Definition 2.2. Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let f be a function from the fuzzy topological space (X,T) to the fuzzy topological space (Y,S). Let λ be a fuzzy set in (Y,S). The inverse image of λ under f written as $f^{-1}(\lambda)$ is the fuzzy set in (X,T) defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in X$. Also the image of λ in (X,T) under f written as $f(\lambda)$ is the fuzzy set in (Y,S) defined by

$$f(\lambda)(y) = \begin{cases} sup_{x \in f^{-1}(y)} & \lambda(x), \quad if\ f^{-1}(y)\ is\ non-empty; \\ 0, & otherwise \end{cases} \qquad \text{for each } y \in Y.$$

Lemma 2.3 ([3]). *Let $f : (X,T) \to (Y,S)$ be a mapping. For fuzzy sets λ and μ of (X,T) and (Y,S) respectively, the following statements hold:*

1. $ff^{-1}(\mu) \leq \mu$;
2. $f^{-1}f(\lambda) \geq \lambda$;
3. $f(1-\lambda) \geq 1 - f(\lambda)$;
4. $f^{-1}(1-\mu) = 1 - f^{-1}(\mu)$;
5. *If f is one-to-one, then* $f^{-1}f(\lambda) = \lambda$;
6. *If f is onto, then* $ff^{-1}(\mu) = \mu$;
7. *If f is one-to-one and onto, then* $f(1-\lambda) = 1 - f(\lambda)$.

Definition 2.4 ([4]). A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$.

Definition 2.5 ([2]). A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy F_σ-set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.6 ([2]). A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy G_δ-set in (X,T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in T$ for $i \in I$.

Definition 2.7 ([5]). A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy σ-nowhere dense if λ is a fuzzy F_σ-set in (X,T) such that $int(\lambda) = 0$.

Lemma 2.8 ([1]). *For a family $\mathscr{A}$ of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space X, $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case $\mathscr{A}$ is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$, where $\alpha \in \mathscr{A}$.*

Definition 2.9 ([4]). A function $f : (X,T) \to (Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is called somewhat fuzzy continuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ implies that there exists a fuzzy open set δ in (X,T) such that $\delta \neq 0$ and $\delta \leq f^{-1}(\lambda)$.

Definition 2.10 ([4]). A function $f : (X,T) \to (Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is called somewhat fuzzy open if $\lambda \in T$ and $\lambda \neq 0$ implies that there exists a fuzzy open set η in (Y,S) such that $\eta \neq 0$ and $\eta \leq f(\lambda)$.

Definition 2.11 ([5]). A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy σ-first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_i's are fuzzy σ-nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy σ-second category.

Definition 2.12 ([5]). Let λ be a fuzzy σ-first category set in a fuzzy topological space (X,T). Then $1-\lambda$ is called a fuzzy σ-residual set in (X,T).

3. Fuzzy σ-Baire spaces

Definition 3.1 ([5]). Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy σ-Baire space if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i's are fuzzy σ-nowhere dense sets in (X,T).

Theorem 3.2 ([5]). *Let (X,T) be a fuzzy topological space. Then the following are equivalent:*

(1) *(X,T) is a fuzzy σ-Baire space.*
(2) *$int(\lambda) = 0$ for every fuzzy σ-first category set λ in (X,T).*
(3) *$cl(\mu) = 1$ for every fuzzy σ-residual set μ in (X,T).*

Definition 3.3 ([6]). A fuzzy topological space (X,T) is called a fuzzy Volterra space if $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where λ_i's are fuzzy dense and fuzzy G_δ-sets in (X,T).

Theorem 3.4 ([5]). *In a fuzzy topological space (X,T), a fuzzy set λ is fuzzy σ-nowhere dense if and only if $1-\lambda$ is a fuzzy dense and fuzzy G_δ-set.*

Proposition 3.5. *If a fuzzy topological space (X,T) is a fuzzy Volterra space, then $int(\vee_{i=1}^{N}(\lambda_i)) = 0$, where λ_i's are fuzzy σ-nowhere dense sets in (X,T).*

Proof. Let λ_i's $[i = 1$ *to* $N]$ be fuzzy σ-nowhere dense sets in (X,T). Then by Theorem 3.4, $(1-\lambda_i)$'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy Volterra space, $cl(\wedge_{i=1}^{N}(1-\lambda_i)) = 1$. Now $cl(\wedge_{i=1}^{N}(1-\lambda_i)) = 1$ implies that $cl(1-\vee_{i=1}^{N}(\lambda_i)) = 1$. Then we have $1 - int(\vee_{i=1}^{N}(1-\lambda_i)) = 1$. This implies that $int(\vee_{i=1}^{N}(\lambda_i)) = 0$. □

Proposition 3.6. *If a fuzzy topological space (X,T) is a fuzzy σ-Baire space, then (X,T) is a fuzzy Volterra space.*

Proof. Let (X,T) be a fuzzy σ-Baire space. Then $int(\vee_{i=1}^{\infty}(\lambda_i) = 0$, where λ_i's are fuzzy σ-nowhere dense sets in (X,T). Now $int(\vee_{i=1}^{N}(\lambda_i)) \leq int(\vee_{i=1}^{\infty}(\lambda_i))$ implies that $int(\vee_{i=1}^{N}(\lambda_i)) = 0$. Then $1 - int(\vee_{i=1}^{N}(\lambda_i)) = 1 - 0 = 1$. Then we have $cl(1 - \vee_{i=1}^{N}(\lambda_i)) = 1$. This implies that $cl(\wedge_{i=1}^{N}(1-\lambda_i)) = 1$. Since λ_i's $[i = 1$ *to* $N]$ are fuzzy σ-nowhere dense sets in (X,T), by Theorem 3.4, $(1-\lambda_i)$'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Hence $cl(\wedge_{i=1}^{N}(1-\lambda)) = 1$, where $(1-\lambda_i)$'s are fuzzy dense and fuzzy G_δ-sets in (X,T) implies that (X,T) is a fuzzy Volterra space. □

4. Fuzzy σ-Baire spaces and functions

Let f be a function from the fuzzy topological space (X,T) to the fuzzy topological space (Y,S). Under what conditions on f may we assert that if (X,T) is a fuzzy σ-Baire space, then (Y,S) is a fuzzy σ-Baire space?.
It may be noticed that the fuzzy continuous image of a fuzzy σ-Baire space may fail to be a fuzzy σ-Baire space. For, consider the following example:

Example 4.1. Let $X = \{a, b, c\}$. The fuzzy sets $\lambda, \mu, \nu, \alpha, \beta$ and η are defined on X as follows:
$\lambda : X \to [0, 1]$ defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$
$\mu : X \to [0, 1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.4$; $\mu(c) = 0.2$
$\nu : X \to [0, 1]$ defined as $\nu(a) = 0.7$; $\nu(b) = 0.5$; $\nu(c) = 0.8$
$\alpha : X \to [0, 1]$ defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.7$; $\alpha(c) = 0.5$
$\beta : X \to [0, 1]$ defined as $\beta(a) = 0.4$; $\beta(b) = 0.2$; $\beta(c) = 0.8$
$\eta : X \to [0, 1]$ defined as $\eta(a) = 0.5$; $\eta(b) = 0.8$; $\eta(c) = 0.7$

Then $T = \{0, \lambda, \mu, \nu, (\lambda \vee \mu), (\lambda \vee \nu), (\mu \vee \nu), (\lambda \wedge \mu), (\lambda \wedge \nu), (\mu \wedge \nu), \lambda \vee (\mu \wedge \nu), \mu \vee (\lambda \wedge \nu), \nu \wedge (\lambda \vee \mu), (\lambda \vee \mu \vee \nu), 1\}$ and $S = \{0, \alpha, \beta, \eta, (\alpha \vee \beta), (\alpha \vee \eta), (\beta \vee \eta), (\alpha \wedge \beta), (\alpha \wedge \eta), (\beta \wedge \eta), \alpha \vee (\beta \wedge \eta), \beta \vee (\alpha \wedge \eta), \eta \wedge (\alpha \vee \beta), 1\}$ are fuzzy topologies on X. Now $1 - [\nu \wedge (\lambda \vee \mu)]$ and $1 - (\mu \wedge \nu)$ are fuzzy σ-nowhere dense sets in (X, T) and $int(\{1 - [\nu \wedge (\lambda \vee \mu)]\} \vee \{1 - (\mu \wedge \nu)\}) = 0$ and hence (X, T) is a fuzzy σ-Baire space. Also the fuzzy topological space (X, S) is not a fuzzy σ-Baire space, since there are no fuzzy σ-nowhere dense sets δ_i in (X, S) such that $int(\vee_{i=1}^{\infty}(\delta_i)) = 0$.

Define a function $f : (X, T) \to (X, S)$ by $f(a) = c; f(b) = a; f(c) = b$. Clearly f is a fuzzy continuous function from the fuzzy σ-Baire space (X, T) to the fuzzy topological space (X, S) which is not a fuzzy σ-Baire space.

It may also be noticed that the image of a fuzzy σ-Baire space under a fuzzy open function may fail to be a fuzzy σ-Baire space. For, consider the following example :

Example 4.2. Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and δ are defined on X as follows :
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.6$; $\lambda(b) = 0.4$; $\lambda(c) = 0.3$
$\mu : X \to [0, 1]$is defined as $\mu(a) = 0.5$; $\mu(b) = 0.7$; $\mu(c) = 0.2$
$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.7$; $\delta(b) = 0.5$; $\delta(c) = 0.6$

Then $T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \mu \vee \delta, \lambda \wedge \mu, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), 1\}$ and $S = \{0, \delta, \lambda \vee \mu, \delta \vee (\lambda \vee \mu), \delta \wedge (\lambda \vee \mu), 1\}$ are fuzzy topologies on X. Now $1 - \delta = (1 - \delta) \vee \{1 - [\delta \vee (\lambda \vee \mu)]\}$ and hence $1 - \delta$ is a fuzzy F_σ-set in (X, S) and $int(1 - \delta) = 0$ and hence $1 - \delta$ is a fuzzy σ- nowhere dense set in (X, S). Also $1 - \{\delta \wedge (\lambda \vee \mu)\} = [1 - (\lambda \vee \mu)] \vee [1 - \{\delta \wedge (\lambda \vee \mu)\}]$ and hence $1 - \{\delta \wedge (\lambda \vee \mu)\}$ is a fuzzy F_σ-set in (X, S) and $int[1 - \{\delta \wedge (\lambda \vee \mu)\}] = 0$ and hence $1 - \{\delta \wedge (\lambda \vee \mu)\}$is a fuzzy σ-nowhere dense set in (X, S). Now $int(\{1 - \delta\} \vee \{1 - [\delta \wedge (\lambda \vee \mu)]\}) = 0$ and hence (X, S) is a fuzzy σ-Baire space.

Now $1 - \lambda = (1 - \lambda) \vee (1 - [\lambda \vee \mu]) \vee (1 - [\lambda \vee (\mu \wedge \delta)])$ and hence $1 - \lambda$ is a fuzzy F_σ-set in (X, T) and $int(1 - \lambda) = 0$ and therefore $1 - \lambda$ is a fuzzy σ-nowhere dense set in (X, T). Also $1 - \mu = (1 - \mu) \vee (1 - [\mu \vee \delta]) \vee (1 - [\lambda \vee \mu])$ and hence $1 - \mu$ is a fuzzy F_σ-set in (X, T) and $int(1 - \mu) = 0$ and therefore $1 - \mu$ is a fuzzy σ-nowhere dense set in (X, T). Now $int(\{1 - \lambda\} \vee \{1 - \mu\}) = \mu \wedge \delta \neq 0$ and hence (X, T) is not a fuzzy σ-Baire space.

Define a function $f : (X, T) \to (X, S)$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Clearly f is an fuzzy open function from the fuzzy σ-Baire space (X, S) to the fuzzy topological space (X, T) which is not a fuzzy σ-Baire space.

Proposition 4.3. *If a function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is fuzzy continuous and fuzzy open*

and if λ is a fuzzy σ-nowhere dense set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy σ- nowhere dense set in (X,T).

Proof. Let λ be a fuzzy σ-nowhere dense set in (Y,S). Then λ is a fuzzy F_σ-set in (Y,S) such that $int(\lambda)=0$. Then $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy closed sets in (Y,S). Now $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$ implies that $f^{-1}(\lambda)=f^{-1}(\vee_{i=1}^{\infty}(\lambda_i))$. Then $f^{-1}(\lambda)=\vee_{i=1}^{\infty}(f^{-1}(\lambda_i))$. Since the function f is fuzzy continuous and λ_i's are fuzzy closed sets in (Y,S), $f^{-1}(\lambda_i)$'s are fuzzy closed sets in (X,T). Hence $f^{-1}(\lambda)$ is a fuzzy F_σ-set in (X,T).

Now we claim that $int(f^{-1}(\lambda))=0$. Suppose that $int(f^{-1}(\lambda))\neq 0$. Then there exists a fuzzy open set μ in (X,T) such that $\mu\leq f^{-1}(\lambda)$. Then $f(\mu)\leq ff^{-1}(\lambda)\leq\lambda$. That is, $f(\mu)\leq\lambda$. Since f is fuzzy open, $f(\mu)$ is a fuzzy open set in (Y,S). This implies that $int(\lambda)\neq 0$ which is a contradiction to $int(\lambda)=0$ in (Y,S). Hence we must have $int(f^{-1}(\lambda))=0$. Therefore $f^{-1}(\lambda)$ is a fuzzy F_σ-set in (X,T) such that $int(f^{-1}(\lambda))=0$ and hence $f^{-1}(\lambda)$ is a fuzzy σ-nowhere dense set in (X,T). □

Proposition 4.4. *If a function $f:(X,T)\to(Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous and fuzzy open and if λ is a fuzzy σ-first category set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy σ-first category set in (X,T).*

Proof. Let λ be a fuzzy σ-first category set in (Y,S). Then $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy σ -nowhere dense sets in (Y,S). Now $f^{-1}(\lambda)=f^{-1}(\vee_{i=1}^{\infty}(\lambda_i))$. Then $f^{-1}(\lambda)=\vee_{i=1}^{\infty}(f^{-1}(\lambda_i))$. Since the function f is fuzzy continuous and fuzzy open and λ_i's are fuzzy σ-nowhere dense sets in (Y,S), by Proposition 4.3, $f^{-1}(\lambda_i)$'s are fuzzy σ-nowhere dense sets in (X,T). Hence $f^{-1}(\lambda)=\vee_{i=1}^{\infty}(f^{-1}(\lambda_i))$, where $f^{-1}(\lambda_i)$'s are fuzzy σ-nowhere dense sets in (Y,S) implies that $f^{-1}(\lambda)$ is a fuzzy σ-first category set in (X,T). □

Theorem 4.5 ([7])**.** *Let $f:(X,T)\to(Y,S)$ be a fuzzy open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then for every fuzzy set β in (Y,S), $f^{-1}(cl(\beta))\leq clf^{-1}(\beta)$.*

Proposition 4.6. *Let $f:(X,T)\to(Y,S)$ be a fuzzy open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then for every fuzzy set δ in (Y,S), $int(f^{-1}(\delta))\leq f^{-1}(int(\delta))$.*

Proof. Let δ be a fuzzy set in (Y,S). Then for the fuzzy set $1-\delta$, we have, by Theorem 4.5, $f^{-1}(cl(1-\delta))\leq clf^{-1}(1-\delta)$. Then $f^{-1}(1-int(\delta))\leq cl(1-f^{-1}(\delta))$. This implies that $(1-f^{-1}(int(\delta))\leq(1-int(f^{-1}(\delta)))$. Hence we have $int(f^{-1}(\delta))\leq f^{-1}(int(\delta))$. □

Proposition 4.7. *If a function $f:(X,T)\to(Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous and fuzzy open and if (Y,S) is a fuzzy σ-Baire space, then (X,T) is a fuzzy σ-Baire space.*

Proof. Let λ be a fuzzy σ-first category set in (Y,S). Then by Proposition 4.4, $f^{-1}(\lambda)$ is a fuzzy σ-first category set in (X,T). Since f is a fuzzy open function, by Proposition 4.6, for the fuzzy set λ in (Y,S), we have $int(f^{-1}(\lambda))\leq f^{-1}(int(\lambda).....(A)$. Since (Y,S) is a fuzzy σ-Baire space, by Theorem 3.2, $int(\lambda)=0$. Then from (A),

we have $int(f^{-1}(\lambda)) \leq f^{-1}(0) = 0$. That is, $int(f^{-1}(\lambda)) = 0$. Hence, by Theorem 3.2, (X, T) is a fuzzy σ-Baire space. □

Proposition 4.8. *If a function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is fuzzy continuous, one-to-one and fuzzy closed and if μ is a fuzzy σ-nowhere dense set in (X, T), then $f(\mu)$ is a fuzzy σ-nowhere dense set in (Y, S).*

Proof. Let μ be a fuzzy σ-nowhere dense set in (X, T). Then μ is a fuzzy F_σ-set in (X, T) such that $int(\mu) = 0$. Then $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where μ_i's are fuzzy closed sets in (X, T). Now $f(\mu) = f(\vee_{i=1}^{\infty}(\mu_i)) = \vee_{i=1}^{\infty}(f(\mu_i))$. Since the function f is fuzzy closed and μ_i's are fuzzy closed sets in (X, T), $f(\mu_i)$'s are fuzzy closed sets in (Y, S). Hence $f(\mu)$ is a fuzzy F_σ-set in (Y, S).

Now we claim that $int(f(\mu)) = 0$. Suppose that $int(f(\mu)) \neq 0$. Then there exists a fuzzy open set η in (Y, S) such that $\eta \leq f(\mu)$. Then $f^{-1}(\eta) \leq f^{-1}f(\mu)$. Since f is one-to-one, $f^{-1}f(\mu) = \mu$. Hence $f^{-1}(\eta) \leq \mu$. Since f is fuzzy continuous and η is a fuzzy open set in (Y, S), $f^{-1}(\eta)$ is a fuzzy open set in (X, T). This implies that $int(\mu) \neq 0$ which is a contradiction to $int(\mu) = 0$ in (X, T). Hence we must have $int(f(\mu)) = 0$. Therefore $f(\mu)$ is a fuzzy F_σ-set in (Y, S) such that $int(f(\mu)) = 0$ and hence $f(\mu)$ is a fuzzy σ-nowhere dense set in (Y, S). □

Proposition 4.9. *If a function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is fuzzy continuous, one-to-one and fuzzy closed and if μ is a fuzzy σ-first category set in (X, T), then $f(\mu)$ is a fuzzy σ-first category set in (Y, S).*

Proof. Let μ be a fuzzy σ-first category set in (X, T). Then $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where μ_i's are fuzzy σ-nowhere dense sets (X, T). Now $f(\mu) = f(\vee_{i=1}^{\infty}(\mu_i)) = \vee_{i=1}^{\infty}(f(\mu_i))$. Since f is a fuzzy continuous, one-to-one and fuzzy closed function and μ_i's are fuzzy σ-nowhere dense sets in (X, T), by Proposition 4.8, $f(\mu_i)$'s are fuzzy σ-nowhere dense sets in (Y, S). Hence $f(\mu)$ is a fuzzy σ-first category set in (Y, S). □

Proposition 4.10. *If a function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is one-to-one, fuzzy continuous and fuzzy closed and if (X, T) is a fuzzy σ-Baire space, then (Y, S) is a fuzzy σ-Baire space.*

Proof. Let μ be a fuzzy σ-first category set in (X, T). Since (X, T) is a fuzzy σ-Baire space, by Theorem 3.2, we have $int(\mu) = 0$. Since f is a fuzzy continuous, one-to-one and fuzzy closed function and μ is a fuzzy σ-first category set in (X, T), by Proposition 4.9, $f(\mu)$ is a fuzzy σ-first category set in (Y, S).

Now we claim that $int(f(\mu)) = 0$. Suppose that $int(f(\mu)) \neq 0$. Then there exists a fuzzy open set η in (Y, S) such that $\eta \leq f(\mu)$. Then we have $f^{-1}(\eta) \leq f^{-1}f(\mu)$. Since f is one-to-one, $f^{-1}f(\mu) = \mu$. Then $f^{-1}(\eta) \leq \mu$. Since f is fuzzy continuous and η is fuzzy open, $f^{-1}(\eta)$ is fuzzy open in (X, T). Hence $int(\mu) \neq 0$ which is a contradiction to $int(\mu) = 0$ in (Y, S). Hence we must have $int(f(\mu)) = 0$. Therefore, for a fuzzy σ-first category set $f(\mu)$ in (Y, S), we have $int(f(\mu)) = 0$. Hence, by Theorem 3.2, (Y, S) is a fuzzy σ-Baire space. □

Theorem 4.11 ([4]). *Suppose (X,T) and (Y,S) are two fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an onto function. Then the following conditions are equivalent:*

(1) *f is somewhat fuzzy open.*
(2) *If λ is a fuzzy dense set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T).*

Proposition 4.12. *If the function $f:(X,T)\to(Y,S)$ from a fuzzy topological space (X,T) onto another fuzzy topological space (Y,S) is somewhat fuzzy open and if $int(\lambda)=0$ in (Y,S), then $int(f^{-1}(\lambda))=0$ in (X,T).*

Proof. Let λ be a fuzzy set in (Y,S) such that $int(\lambda)=0$. Then $1-int(\lambda)=1$ implies that $cl(1-(\lambda))=1$. That is, $1-(\lambda)$ is a fuzzy dense set in (Y,S). Since f is somewhat fuzzy open, by Theorem 4.11,$f^{-1}(1-(\lambda))$ is a fuzzy dense set in (X,T). That is, $cl(f^{-1}(1-(\lambda)))=1$. This implies that $cl(1-f^{-1}(\lambda))=1$. Then $1-int(f^{-1}(\lambda))=1$. That is, $int(f^{-1}(\lambda))=0$. □

Proposition 4.13. *If the function $f:(X,T)\to(Y,S)$ from a fuzzy topological space (X,T) onto another fuzzy topological space (Y,S) is fuzzy continuous and if λ is a fuzzy F_σ-set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy F_σ-set in (X,T).*

Proof. Let λ be a fuzzy F_σ-set in (Y,S). Then $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy closed sets in (Y,S). Now $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$, implies that $f^{-1}(\lambda)=f^{-1}(\vee_{i=1}^{\infty}(\lambda_i))$. Then $f^{-1}(\lambda)=\vee_{i=1}^{\infty}(f^{-1}(\lambda_i))$. Since the function f is fuzzy continuous and λ_i's are fuzzy closed sets in (Y,S), $f^{-1}(\lambda_i)$'s are fuzzy closed sets in (X,T). Hence $f^{-1}(\lambda)$ is a fuzzy F_σ-set in (X,T). □

Proposition 4.14. *If the function $f:(X,T)\to(Y,S)$ from a fuzzy topological space (X,T) onto another fuzzy topological space (Y,S) is a fuzzy continuous and somewhat fuzzy open function and if λ is a fuzzy σ-nowhere dense set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy σ-nowhere dense set in (X,T).*

Proof. Let λ be a fuzzy σ-nowhere dense set in (Y,S). Then λ is a fuzzy F_σ-set in (Y,S) such that $int(\lambda)=0$. Since f is fuzzy continuous, by Proposition 4.13, $f^{-1}(\lambda)$ is a fuzzy F_σ-set in (X,T). Since f is somewhat fuzzy open, by Proposition 4.12, $int(f^{-1}(\lambda))=0$. Hence $f^{-1}(\lambda)$ is a fuzzy σ-nowhere dense set in (X,T). □

Proposition 4.15. *If the function $f:(X,T)\to(Y,S)$ from a fuzzy topological space (X,T) onto another fuzzy topological space (Y,S) is a fuzzy continuous and somewhat fuzzy open function and if λ is a fuzzy σ-first category set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy σ-first category set in (X,T).*

Proof. Let λ be a fuzzy σ-first category set in (Y,S). Then $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy σ-nowhere dense sets in (Y,S). Now $f^{-1}(\lambda)=f^{-1}(\vee_{i=1}^{\infty}(\lambda_i))=\vee_{i=1}^{\infty}(f^{-1}(\lambda_i))$. Since f is a fuzzy continuous, somewhat fuzzy open function and λ_i's are fuzzy σ-nowhere dense sets $(Y,S),byProposition4.14,f^{-1}(\lambda_i)$'s are fuzzy σ-nowhere dense sets in (X,T). Hence $f^{-1}(\lambda)$ is a fuzzy σ-first category set in (X,T). □

Proposition 4.16. *If the function $f:(X,T)\to(Y,S)$ from a fuzzy topological space (X,T) onto another fuzzy topological space (Y,S) is a fuzzy continuous and*

somewhat fuzzy open function and if (Y, S) *is a fuzzy* σ*-Baire space, then* (X, T) *is a fuzzy* σ*-Baire space.*

Proof. Let λ be a fuzzy σ-first category set in (Y, S). Since (Y, S) is a fuzzy σ-Baire space, by Theorem 3.2, $int(\lambda) = 0$ in (Y, S). Since f is a fuzzy continuous and somewhat fuzzy open function, by Proposition 4.15, $f^{-1}(\lambda)$ is a fuzzy σ-first category set in (X, T).

Now we claim that $int(f^{-1}(\lambda)) = 0$. Suppose that $int(f^{-1}(\lambda)) \neq 0$. Then there exists a fuzzy open set μ in (X, T) such that $\mu \leq f^{-1}(\lambda)$. Then $f(\mu) \leq ff^{-1}(\lambda) \leq \lambda$. That is, $f(\mu) \leq \lambda$. Then $int(f(\mu)) \leq int(\lambda)$. Since f is fuzzy somewhat open and μ is a fuzzy open set in $(X, T), int(f(\mu)) \neq 0$. This implies that $int(\lambda) \neq 0$ which is a contradiction to $int(\lambda) = 0$ in (Y, S). Hence we must have $int(f^{-1}(\lambda)) = 0$. Hence, by Theorem 3.2, (X, T) is a fuzzy σ-Baire space. □

Theorem 4.17 ([4])**.** *Let* (X, T) *and* (Y, S) *be any two fuzzy topological spaces. Let* $f : (X, T) \to (Y, S)$ *be a function. Then the following are equivalent:*

(1) *f is somewhat fuzzy continuous.*
(2) *If* λ *is a fuzzy closed set of* (Y, S) *such that* $f^{-1}(\lambda) \neq 1$*, then there exists a proper fuzzy closed set* μ *of* (X, T) *such that* $\mu > f^{-1}(\lambda)$.
(3) *If* λ *is a fuzzy dense set in* (X, T)*, then* $f(\lambda)$ *is a fuzzy dense set in* (Y, S).

Proposition 4.18. *If the function* $f : (X, T) \to (Y, S)$ *from a fuzzy topological space* (X, T) *onto another fuzzy topological space* (Y, S) *is somewhat fuzzy continuous one-to-one and if* $int(\lambda) = 0$ *in* (X, T)*, then* $int(f(\lambda)) = 0$ *in* (Y, S).

Proof. Let λ be a fuzzy set in (X, T) such that $int(\lambda) = 0$. Then $1 - int(\lambda) = 1$ implies that $cl(1 - (\lambda)) = 1$. That is, $1 - (\lambda)$ is a fuzzy dense set in (X, T). Since f is somewhat fuzzy continuous, by Theorem 4.17, $f(1-(\lambda))$ is a fuzzy dense set in (Y, S). That is, $cl(f(1-(\lambda))) = 1$. Since f is one-to-one and onto, $f(1-\lambda) = 1-f(\lambda)$. Then $cl(1 - f(\lambda)) = 1$ and hence $1 - int(f(\lambda)) = 1$. That is, $int(f(\lambda)) = 0$ in (Y, S). □

Proposition 4.19. *If a function* $f : (X, T) \to (Y, S)$ *from a fuzzy topological space* (X, T) *into another fuzzy topological space* (Y, S) *is somewhat fuzzy continuous, one-to-one, onto and fuzzy closed and if* μ *is a fuzzy* σ*-nowhere dense set in* (X, T)*, then* $f(\mu)$ *is a fuzzy* σ*-nowhere dense set in* (Y, S).

Proof. Let μ be a fuzzy σ-nowhere dense set in (X, T). Then μ is a fuzzy F_σ-set in (X, T) such that $int(\mu) = 0$. Then $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where μ_i's are fuzzy closed sets in (X, T). Now $f(\mu) = f(\vee_{i=1}^{\infty}(\mu_i)) = \vee_{i=1}^{\infty}(f(\mu_i))$. Since the function f is fuzzy closed and μ_i's are fuzzy closed sets in (X, T), $f(\mu_i)$'s are fuzzy closed sets in (Y, S) .Hence $f(\mu)$ is a fuzzy F_σ-set in (Y, S). Since the function f is somewhat fuzzy continuous, one-to-one, onto and since $int(\mu) = 0$ in (X, T), by Proposition 4.18 , we have $int(f(\mu)) = 0$ in (Y, S). Therefore $f(\mu)$ is a fuzzy F_σ-set in (Y, S) such that $int(f(\mu)) = 0$ and hence $f(\mu)$ is a fuzzy σ-nowhere dense set in (Y, S). □

Proposition 4.20. *If a function* $f : (X, T) \to (Y, S)$ *from a fuzzy topological space* (X, T) *into another fuzzy topological space* (Y, S) *is somewhat fuzzy continuous, one-to-one, onto and fuzzy closed and if* μ *is a fuzzy* σ*-first category set in* (X, T)*, then* $f(\mu)$ *is a fuzzy* σ*-first category set in* (Y, S).

Proof. Let μ be a fuzzy σ-first category set in (X,T). Then $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where μ_i's are fuzzy σ-nowhere dense sets (X,T). Now $f(\mu) = f(\vee_{i=1}^{\infty}(\mu_i)) = \vee_{i=1}^{\infty}(f(\mu_i))$. Since f is a somewhat fuzzy continuous, one-to-one and fuzzy closed function and μ_i's are fuzzy σ-nowhere dense sets in (X,T), by Proposition 4.19, $f(\mu_i)$'s are fuzzy σ-nowhere dense sets in (Y,S). Hence $f(\mu)$ is a fuzzy σ-first category set in (Y,S). □

Proposition 4.21. *If a function $f : (X,T) \to (Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is somewhat fuzzy continuous, one-to-one, onto and fuzzy closed and if (X,T) is a fuzzy σ-Baire space , then (Y,S) is a fuzzy σ-Baire space.*

Proof. Let μ be a fuzzy σ-first category set in (X,T). Since (X,T) is a fuzzy σ-Baire space, by Theorem 3.2, we have $int(\mu) = 0$. Since f is a somewhat fuzzy continuous, one-to-one and fuzzy closed function and μ is a fuzzy σ-first category set in (X,T), by Proposition 4.20, $f(\mu)$ is a fuzzy σ-first category set in (Y,S).

Now we claim that $int(f(\mu)) = 0$. Suppose that $int(f(\mu)) \neq 0$. Then there exists a fuzzy open set η in (Y,S) such that $\eta \leq f(\mu)$. Then we have $f^{-1}(\eta) \leq f^{-1}f(\mu)$. Since f is one-to-one, $f^{-1}f(\mu) = \mu$. Then $f^{-1}(\eta) \leq \mu$. Since f is somewhat fuzzy continuous and η is fuzzy open, there exists a fuzzy open set δ in (X,T) such that $\delta \leq f^{-1}(\eta)$. Then $\delta \leq \mu$.Hence $int(\mu) \neq 0$ which is a contradiction to $int(\mu) = 0$ in (X,T). Hence we must have $int(f(\mu)) = 0$. Therefore, for a fuzzy σ-first category set $f(\mu)$ in (Y,S), we have $int(f(\mu)) = 0$. Hence, by Theorem 3.2, (Y,S) is a fuzzy σ-Baire space. □

5. Conclusions

In this paper, the interrelation between fuzzy σ-Baire spaces and fuzzy Volterra spaces are studied. Some results concerning fuzzy closed, fuzzy continuous, somewhat fuzzy continuous and somewhat fuzzy open functions that preserve σ-Baire spaces in the context of images and pre-images are established.

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On Fuzzy Pre-σ-Baire Spaces

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ABSTRACT

In this paper, the concepts offuzzy preσ-nowheredense set, fuzzy pre σ-first categoryset and fuzzy preσ-second categoryset in fuzzy topological spaces, are introduced and studied. By means of fuzzy preσ-nowhere dense sets, the concept of fuzzypre σ-Baire space is defined and several characterizationsof fuzzy pre σ-Baire spacesare studied. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: Fuzzy pre-closed set, fuzzy pre F_σ-set, fuzzy pre G_δ-set, fuzzy nowheredenseset, fuzzy preσ-second category set, fuzzy pre σ-Baire space, fuzzy pre Baire space.

2000 AMS Classification: 54 A 40, 03 E 72.

1. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by **L. A. Zadeh** [15] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. Among the first fields of Mathematics to be considered in the context of fuzzy sets was general topology. The concepts of fuzzy topology was defined by **C. L. Chang** [6] in the year 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then, much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

The concepts of Baire spaces have been studied extensively in classical topology **[7],** [**8**] and [**10**]. The concept of fuzzy σ –nowhere dense sets in classical topology, was introduced and studied By **Jiling Cao** and**Sina Greenwood** [**9**]. This concept in fuzzy

setting was introduced and studied by **G. Thangaraj**and **E. Poongothai[14]**and by means of fuzzy σ −nowhere dense sets, the concept of fuzzy σ –Baire spaces was introduced and studied in [14]. **A. S. Bin Shahna** [**5**] introduced the concept of fuzzy pre-open sets in fuzzy topological spaces. In this paper, the concepts of fuzzypre σ − nowhere dense sets are introduced and studied. By means of fuzzy pre σ −nowhere dense sets, the concept of fuzzy pre σ −Baire space isintroduced and several characterizations of fuzzy pre σ − Baire spaces are studied. The inter-relationshipbetween pre σ − Baire spaces and fuzzy σ − Baire spaces havebeen established in thispaper.

2. PRELIMINARIES

In order to make the exposition self-contained, we give some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to CHANG. Let X be a non-empty set and I the unit interval [0, 1]. A fuzzy set λ in X is a mapping from X into I.

Definition 21 [6]: Let λ and μ be fuzzy sets in X. Then, for all $x \in X$,

(i) $\lambda = \mu \iff \lambda(x) = \mu(x)$

(ii) $\lambda \leq \mu \iff \lambda(x) \leq \mu(x)$

(iii) $\psi = \lambda \vee \mu \iff \psi(x) = \max\{\lambda(x), \mu(x)\}$

(iv) $\delta = \lambda \wedge \mu \iff \psi(x) = \min\{\lambda(x), \mu(x)\}$

(v) $\eta = \lambda^c \iff \eta(x) = 1 - \lambda(x)$

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$, are defined respectively as

(vi) $\psi(x) = sup_i\{\lambda_i(x) / x \in X\}$

(vii) $\delta(x) = inf_i\{\lambda_i(x) / x \in X\}$

Definition 2. 2[6]: Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). Thefuzzy interior and the fuzzyclosure of a fuzzy set λ are defined respectivelyas follows:

(i). int (λ) = { μ| μ ≤ λ, μ ∈ T}

(ii). cl (λ) = { μ|λ ≤ μ, 1−μ ∈ T }.

Lemma 2. 1 [1]: For a fuzzy set λ of a fuzzy topological space X, (i). $1 -$ Int $(\lambda) =$ Cl $(1 - \lambda)$ and (ii). $1 -$ Cl (λ) = Int $(1 - \lambda)$.

Definition 2. 3 [12]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, cl $(\lambda) = 1$, in (X, T).

Definition 2. 4 [12]: A fuzzy set λ ina fuzzy topological space (X, T) is calledfuzzy nowhere dense if thereexists no non-zero fuzzy open set μ in (X, T) such that $\mu <$ cl (λ). That is, int cl (λ) = 0, in (X, T).

Definition 2. 5 [5]: A fuzzy set λ in a fuzzy topological space(X, T) is called
(i). a fuzzy pre-open setin (X, T) ifλ ≤intcl (λ),
(ii). a fuzzy pre-closed set in (X, T) ifcl int (λ) ≤ λ.

Definition 2. 6 [5]: Let (X, T) be afuzzytopological space. The fuzzy pre-interior and the fuzzy pre-closure of a fuzzy set λ, are defined respectivelyas follows:
(i) . $\text{pint}(\lambda) = \{ \mu | \mu \leq \lambda, \mu \text{ is a fuzzy pre-open set of X} \}$;
(ii) $\text{pcl}(\lambda) = \{ \mu | \lambda \leq \mu, \mu \text{ is a fuzzy pre-closed set of X} \}$.

Definition 2. 7 [5]: A fuzzy setλin a fuzzy topological space (X, T) is calleda fuzzy pre-closed set if $\lambda = \text{pcl}(\lambda)$and fuzzy pre-open set if$\lambda = \text{pint}(\lambda)$.

Lemma2. 2 [5]: Letλbe afuzzy set of a fuzzy topological space (X, T). Then, (i). $1 - \text{pcl}(\lambda) = \text{pint}(1 - \lambda)$ and (ii). $1 - \text{pint}(\lambda) = \text{pcl}(1 - \lambda)$.

Definition 2. 8 [2]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ-set in (X, T) if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $(1 - \lambda_i) \in T$ for $i \in I$.

Definition 2. 9 [2]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy G_δ-set in (X, T) if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in T$ for $i \in I$.

Definition 2. 10 [8]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzyσ −first category if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy σ −nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said tobe of fuzzyσ −second category in (X, T).

Theorem 2. 1[4]: Let λbe a fuzzy set of a fuzzy topological space (X, T). Then, $(i).\text{pcl}(\lambda) \geq \lambda \wedge \text{cl int}(\lambda); (ii).\text{pint}(\lambda) \leq \lambda \wedge \text{int cl}(\lambda)$.

Theorem 2. 2[4]: Let λbe a fuzzy set of a fuzzy topological space(X, T). Then, the following properties hold:
(1) $\text{int}\big(\text{pcl}(\lambda)\big) \leq \text{int cl}(\lambda)$;
(2) $\text{int}\big(\text{pcl}(\lambda)\big) \geq \text{int}\big(\text{cl int}(\lambda)\big)$.

Theorem 2. 3[4]: Let λbe a fuzzy set of a fuzzy topological space(X, T). Then, $\text{int}(\lambda) \leq \text{pint}(\lambda) \leq \lambda \leq \text{pcl}(\lambda) \leq \text{cl}(\lambda)$.

Lemma 2. 3 [4]: Let λ and μ be fuzzysets of a fuzzy topological space (X, T). Then,
(i). $\text{pcl}(\lambda \vee \mu) \geq \text{pcl}(\lambda) \vee \text{pcl}(\mu)$;
(ii) . $\text{pcl}(\lambda \wedge \mu) \leq \text{pcl}(\lambda) \wedge \text{pcl}(\mu)$.

Remarks2. 1: From the abovelemmaone can easily establish the following properties:
(1) . $\text{pint}(\lambda \vee \mu) \geq \text{pint}(\lambda) \vee \text{pint}(\mu)$;
(2) . $\text{pint}(\lambda \wedge \mu) \leq \text{pint}(\lambda) \wedge \text{pint}(\mu)$.

Definition2. 11 [14]: Let (X, T) be a fuzzy topological space. A fuzzy set λin (X, T) is called a fuzzy pre nowhere dense set if there exists no non-zero fuzzy pre-open setμin (X, T)such that $\mu < pcl(\lambda)$. That is, $\text{pint}\,\text{pcl}(\lambda) = 0$, in (X, T).

3. FUZZY PRE σ −NOWHERE DENSE SETS

Definition 3. 1: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy pre F_σ-set in (X, T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$ are fuzzy pre-closed sets in (X, T).

Definition 3. 2: A fuzzy setλ in a fuzzy topological space (X, T) is called a fuzzy pre G_δ-setin (X, T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'$sare fuzzy pre-opensets in (X, T).

Definition 3. 3: A fuzzy set λin a fuzzy topological space (X, T) is called a fuzzy preσ −nowhere dense set if λ is a non-zero fuzzy pre F_σ-set in (X, T) such that pint(λ)=0.

Definition 3. 4: A fuzzy setλ in a fuzzy topological space(X, T) is calledfuzzy pre dense if there exists no fuzzy pre-closed set μin (X, T) such that$\lambda < \mu < 1$. That is, pcl (λ) = 1, in (X, T).

Example 3. 1: Let X={a, b, c}. The fuzzy sets λ, μ and νare defined on X as follows:
$\lambda: X \to [0,1]$is defined as$\lambda(a) = 0.4;\ \lambda(b) = 0.7;\ \lambda(c) = 0.4$.
$\mu: X \to [0,1]$is defined as $\mu(a) = 0.5;\ \mu(b) = 0.4;\ \mu(c) = 0.8$.
$\nu: X \to [0,1]$ is defined as $\nu(a) = 0.2;\ \nu(b) = 0.4;\ \nu(c) = 0.6$.
Then T= $\{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \vee \nu, \lambda \wedge \mu, \lambda \wedge \nu, \nu \vee (\lambda \wedge \nu), 1\}$ is clearly a fuzzy topology on X. Now consider the fuzzy set$\alpha = [1 - (\lambda \vee \mu)] \vee [1 - (\lambda \vee \nu)]$ in (X, T). Then, α is a fuzzy pre F_σ-set in (X, T) and $\text{pint}(\alpha) = 0$ and hence α is a fuzzy pre σ −nowhere dense set in (X, T). The fuzzy set $\beta = (1 - \lambda) \vee (1 - \mu) \vee (1 - \nu)$, is a fuzzy pre F_σ-set in (X, T) and $\text{pint}(\beta) \neq 0$, impliesthat β is not a fuzzy preσ −nowhere dense set in (X, T).

Remarks 3. 1: If λ andμ arefuzzy pre σ − nowhere dense sets in afuzzy topologicalspace(X, T), then$\lambda \vee \mu$need not bea fuzzy pre σ −nowhere dense set in (X, T). For, consider the following example:

Example 3. 2: Let X={a, b, c}. The fuzzy sets λ, μ and νare defined on X as follows:
$\lambda: X \to [0,1]$ isdefined as$\lambda(a) = 0.7;\ \lambda(b) = 0.7;\ \lambda(c) = 0.5$.
$\mu: X \to [0,1]$is defined as $\mu(a) = 0.5;\ \mu(b) = 0.4;\ \mu(c) = 0.8$.
$\nu: X \to [0,1]$isdefined as $\nu(a) = 0.7;\ \nu(b) = 0.6;\ \nu(c) = 0.5$.
Now $\alpha = [(1 - \mu) \vee \{1 - (\lambda \vee \mu)\} \vee \{1 - (\mu \vee \nu)\}]$ and$\beta = [(1 - \lambda) \vee (1 - \nu)]$ are fuzzy pre F_σ-sets in (X, T). Also $\text{pint}(\alpha) = 0$ and $\text{pint}(\beta) = 0$. Therefore α and β are fuzzy preσ −nowhere dense sets in (X, T). Clearly $(\alpha \vee \beta)$ is a fuzzy pre F_σ-set in (X, T). On computation, $\text{pint}(\alpha \vee \beta) = (\mu \wedge \nu) \neq 0$, impliesthat $(\alpha \vee \beta)$ is not a fuzzy pre σ −nowhere dense set in (X, T).

Proposition 3. 1: Afuzzy set λ is afuzzypre $\sigma-$ nowhere dense set in a fuzzy topologicalspace (X, T)if and only if $(1-\lambda)$ is a fuzzy pre dense and fuzzypre G_δ-setin (X, T).

Proof: Let λ be a fuzzy pre $\sigma-$nowhere dense set (X, T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where$(\lambda_i)'s$ are fuzzy pre-closed setsin (X, T) and $\text{pint}(\lambda) = 0$. Then $1-\text{pint}(\lambda) = 1-0 = 1$ and hence by lemma 2. 2, $\text{pcl}(1-\lambda) = 1$. Also$(1-\lambda) = 1-\vee_{i=1}^{\infty}(\lambda_i) = \wedge_{i=1}^{\infty}(1-\lambda_i)$ where $(1-\lambda_i)'$sare fuzzy pre-open sets in (X, T) implies that $1-\lambda$ is a fuzzy pre G_δ-set in (X, T). Hence, $(1-\lambda)$ is a fuzzy pre dense and fuzzy pre G_δ-set in (X, T).
Conversely, let λ bea fuzzy pre denseand fuzzypre G_δ-set in (X, T). Then, $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy pre-open sets in (X, T). Now $1-\lambda = 1-\wedge_{i=1}^{\infty}(\lambda_i) = \vee_{i=1}^{\infty}(1-\lambda_i)$, where $(1-\lambda_i)'s$ are fuzzy pre-closed sets in (X, T). Hence $(1-\lambda)$ is a fuzzy pre F_σ-set in (X, T)and $\text{pint}(1-\lambda) = 1-\text{pcl}(\lambda) = 1-1 = 0$. [since$\lambda$is a fuzzy pre densein(X, T)]. Therefore $(1-\lambda)$is afuzzy pre $\sigma-$nowhere dense set in (X, T).

Proposition 3. 2: If λ is a fuzzy pre dense set in a fuzzy topological space (X, T) such that $\mu \leq (1-\lambda)$, where μ is a fuzzy pre F_σ-set in (X, T), then μ is a fuzzy pre $\sigma-$nowhere dense set in (X, T).

Proof: Let λ be a fuzzy pre dense set in (X, T) such that$\mu \leq (1-\lambda)$. Now $\mu \leq (1-\lambda)$, implies that $\text{pint}(\mu) \leq \text{pint}(1-\lambda)$. Then $\text{pint}(\mu) \leq 1-\text{pcl}(\lambda) = 1-1 = 0$ and hence $\text{pint}(\mu) = 0$. Therefore, μ is a fuzzy pre F_σ-set in (X, T) such that $\text{pint}(\mu) = 0$ and hence $\mu$is a fuzzy pre $\sigma-$nowhere dense set in (X, T).

Proposition 3. 3: If λ is a fuzzy pre F_σ-set and fuzzy pre nowhere dense set in (X, T), thenλ isa fuzzy pre $\sigma-$nowhere dense set in (X, T).

Proof: Now $\lambda \leq \text{pcl}(\lambda)$, for any fuzzy set λ in (X, T). Then, $\text{pint}(\lambda) \leq \text{pintpcl}(\lambda)$. Since λ is a fuzzy pre nowhere dense set in (X, T), $\text{pintpcl}(\lambda) = 0$ and hence $\text{pint}(\lambda) = 0$. Since λ is a fuzzy pre F_σ-set such that $\text{pint}(\lambda) = 0, \lambda$ is afuzzy pre $\sigma-$nowhere dense set in (X, T).

Definition3. 5: Let (X, T) be a fuzzy topological space. A fuzzysetλin (X, T is called fuzzy pre $\sigma-$first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy pre $\sigma-$nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy pre $\sigma-$second category in (X, T).

Definition 3. 6: Let λ be a fuzzy pre $\sigma-$first category set in (X, T). Then $(1-\lambda)$iscalled a fuzzy pre $\sigma-$residual set in (X, T).

Definition 3. 7: A fuzzy topological space (X, T) is called fuzzy pre $\sigma-$first category if the fuzzy set 1_X is a fuzzy pre $\sigma-$first categoryset in (X, T). That is, $1_X =$

$\vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzypre $\sigma-$nowhere dense sets in (X, T). Otherwise, (X, T) will becalled a fuzzy pre $\sigma-$second category space.

Proposition 3. 4: If $\lambda$is a fuzzy pre $\sigma-$first category set in a fuzzy topological space (X, T), then there is a fuzzy pre F_σ-setδin (X, T) such that$\lambda \leq \delta$.

Proof: Letλ be a fuzzy pre $\sigma-$first category set in (X, T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$are fuzzy pre $\sigma-$nowhere dense sets in (X, T). Now $[1-pcl(\lambda_i)]'s$(i=1 to ∞) are fuzzy pre open sets in (X, T). Then $\mu = \wedge_{i=1}^{\infty}(1-pcl(\lambda_i))$ is a fuzzy pre G_δ-set in (X, T) and $1-\mu = 1-[\wedge_{i=1}^{\infty}(1-pcl(\lambda_i))] = [\vee_{i=1}^{\infty} pcl(\lambda_i)]$. Now $(\lambda_i) \leq pcl(\lambda_i)$, implies that $\vee_{i=1}^{\infty}(\lambda_i) \leq [\vee_{i=1}^{\infty} pcl(\lambda_i)]$. Hence$\lambda = \vee_{i=1}^{\infty}(\lambda_i) \leq [\vee_{i=1}^{\infty} pcl(\lambda_i)] = [1-\mu]$. That is, $\lambda \leq [1-\mu]$and$[1-\mu]$is a fuzzy pre F_σ-set in (X, T). Let $\delta = [1-\mu]$. Hence, if λ is a fuzzy pre $\sigma-$first category set in(X, T), then there is a fuzzy pre F_σ-setδin (X, T)such that$\lambda \leq \delta$.

Proposition 3. 5: If $\lambda$is a fuzzy pre $\sigma-$first category set ina fuzzy topological space (X, T), then there is a fuzzy pre F_σ-set δin (X, T) such that $\lambda \leq \delta \leq cl(\lambda)$, where$\delta$ is afuzzy pre F_σ-set in (X, T).

Proof: Let λ be a fuzzy pre $\sigma-$ first category set in (X, T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy pre $\sigma-$nowhere dense sets in (X, T). Now $[1-pcl(\lambda_i)]'s$(i=1 to ∞) are fuzzy pre open sets in (X, T). Then, $\mu = \wedge_{i=1}^{\infty}(1-pcl(\lambda_i))$ is a fuzzy pre G_δ-set in (X, T) and $1-\mu = 1-[\wedge_{i=1}^{\infty}(1-pcl(\lambda_i))] = [\vee_{i=1}^{\infty} pcl(\lambda_i)]$. Now $\lambda = \vee_{i=1}^{\infty}(\lambda_i) \leq [\vee_{i=1}^{\infty} pcl(\lambda_i)] \leq \vee_{i=1}^{\infty}(cl(\lambda_i)) \leq cl(\vee_{i=1}^{\infty}(\lambda_i))$ That is, $\lambda \leq [1-\mu] \leq cl(\lambda)$ and$[1-\mu]$is a fuzzy pre F_σ-set in (X, T). Let $\delta = [1-\mu]$. Hence, if λ is a fuzzy pre $\sigma-$first categoryset in (X, T), then there is a fuzzy pre F_σ-setδin (X, T) such that$\lambda \leq \delta \leq cl(\lambda)$, where$\delta$ is a fuzzy pre F_σ-set in (X, T).

Proposition 3. 6: If the fuzzypre dense sets arefuzzy pre open sets in a fuzzy topological space (X, T) and ifλ is a fuzzy pre $\sigma-$first category set in (X, T), thenλ is a fuzzy pre first category set in (X, T).

Proof: Let $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$be a fuzzy pre $\sigma-$first category set in (X, T), where the fuzzy sets $(\lambda_i)'s$ are fuzzy pre $\sigma-$nowhere dense sets in (X, T). Then $pint(\lambda_i) = 0$ and $(\lambda_i)'s$are fuzzy pre F_σ-sets in (X, T). Now $pint(\lambda_i) = 0$ implies that $1-pint(\lambda_i) = 1-0 = 1$ and hence $pcl(1-\lambda_i) = 1$. By hypothesis, the fuzzy pre dense sets$(1-\lambda_i)'s$ are fuzzy preopen sets in (X, T) and hence $(\lambda_i)'s$ are fuzzy pre closed sets in (X, T). Then, $pcl(\lambda_i) = \lambda_i$ and $pint(\lambda_i) = 0$ implies thatpintpcl$(\lambda_i) = pint(\lambda_i) = 0$. That is, $(\lambda_i)'s$ are fuzzy pre nowhere dense sets in (X, T). Therefore $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, is a fuzzy pre first category set in (X, T).

Proposition 3. 7: If$\lambda$is a fuzzy pre closed set in a fuzzy topological space (X, T) and if $\text{pint}(\lambda) = 0$, then λ is a fuzzy pre $\sigma -$ nowhere dense set in (X, T).

Proof: Let λbe a fuzzy pre closed set in (X, T). Then we have $\text{pcl}(\lambda) = \lambda$. Now $\text{pint}[\text{pcl}(\lambda)] = \text{pint}(\lambda)$ and $\text{pint}(\lambda) = 0$, implies thatλ is a fuzzy pre nowhere dense set in (X, T).

Proposition 3. 8: If λ is a fuzzy closed and fuzzy$\sigma -$ nowhere dense set in a fuzzy topological space(X, T), then$\text{pint}(\lambda) = 0$, in (X, T).

Proof: Letλ be a fuzzy $\sigma -$ nowhere dense set in (X, T). Thenλ is a fuzzy F_σ-set such that $\text{int}(\lambda) = 0$. By theorem 2. 1, we have$\text{pint}(\lambda) \leq \lambda \wedge \text{int cl}(\lambda)$. Then, $\text{pint}(\lambda) \leq \lambda \wedge \text{int}(\lambda)$[since$\lambda$ is a fuzzyclosed set, $\lambda = \text{cl}(\lambda)$] and hence$\text{pint}(\lambda) \leq \lambda \wedge 0$. That is, $\text{pint}(\lambda) = 0$, in (X, T).

Proposition 3. 9: If the fuzzy setsλandμ are fuzzy pre $\sigma -$ nowhere dense sets in a fuzzy topological space (X, T), then$(\lambda \wedge \mu)$ is a fuzzy pre $\sigma -$nowhere dense set in (X, T).

Proof: Let $\lambda$and μbe fuzzy pre $\sigma -$ nowhere dense sets in (X, T). Then, λandμ are fuzzy pre F_σ-sets and $\text{pint}(\lambda) = 0, \text{pint}(\mu) = 0$. Since λandμ are fuzzy pre F_σ-sets in (X, T), $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$ are fuzzy pre-closed sets in (X, T) and$\mu = \vee_{i=1}^{\infty} \mu_j$, where $(\mu_j)'s$ are fuzzy pre-closed sets in (X, T). Now$\lambda \wedge \mu = (\vee_{i=1}^{\infty} \lambda_i) \wedge (\vee_{i=1}^{\infty} \mu_j) = \vee_{i,j=1}^{\infty} (\lambda_i \wedge \mu_j)$. Let $\delta_{ij} = (\lambda_i \wedge \mu_j)$. Then $\lambda \wedge \mu = \vee_{i,j=1}^{\infty} (\delta_{ij})$. Since$\lambda_i$ and μ_j are fuzzy pre closed sets in (X, T), $\text{pcl}(\lambda_i) = \lambda_i$ and $\text{pcl}(\mu_j) = \mu_j$. Then, $\text{pcl}\left(\lambda_i \wedge \mu_j\right) \leq \text{pcl}(\lambda_i) \wedge \text{pcl}(\mu_j) = \lambda_i \wedge \mu_j$. That is, $\text{pcl}(\lambda_i \wedge \mu_j) \leq \lambda_i \wedge \mu_j \ldots\ldots(1)$. But $\lambda_i \wedge \mu_j \leq \text{pcl}(\lambda_i \wedge \mu_j)\ldots\ldots(2)$. From (1) and (2), $\text{pcl}(\lambda_i \wedge \mu_j) = \lambda_i \wedge \mu_j$ and hence$\lambda_i \wedge \mu_j$ is a fuzzy pre closed set in (X, T). This implies that$\delta_{ij}$is a fuzzy pre closed set in (X, T). Therefore $(\lambda \wedge \mu)$ is a fuzzy pre F_σ-set in (X, T). Also $\text{pint}(\lambda \wedge \mu) \leq \text{pint}(\lambda) \wedge \text{pint}(\mu) = 0 \wedge 0 = 0$. Therefore $\text{pint}(\lambda \wedge \mu) = 0$. Hence $(\lambda \wedge \mu)$ is a fuzzy pre F_σ-set such that $\text{pint}(\lambda \wedge \mu) = 0$. Therefore $(\lambda \wedge \mu)$is a fuzzy pre $\sigma -$ nowhere dense set in (X, T).

Proposition 3. 10: If each fuzzy $\sigma -$ nowhere dense set $\lambda$is a fuzzy closed setin a fuzzy topological space (X, T), thenλ is a fuzzy pre $\sigma -$ nowhere dense set in (X, T).

Proof: Let λbe a fuzzy $\sigma -$ nowhere dense set in (X, T). Then λ is a fuzzyF_σ-setin (X, T) suchthat $\text{int}(\lambda) = 0$. By theorem 2. 1, $\text{pint}(\lambda) \leq \lambda \wedge \text{intcl}(\lambda)$. Since$\lambda$ is a fuzzy closed set in (X, T), $\text{cl}(\lambda) = \lambda$. Then $\text{pint}(\lambda) \leq \lambda \wedge \text{int}(\lambda)$. That is, $\text{pint}(\lambda) \leq \lambda \wedge 0 = 0$. Hence, $\text{pint}(\lambda) = 0$ and therefore λ is a fuzzy pre $\sigma -$ nowhere dense set in (X, T).

4. FUZZY PRE σ – BAIRE SPACES

Definition 4. 1: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy pre σ –Baire spaceif $\text{pint}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where$(\lambda_i)'$sare fuzzy pre σ –nowhere dense sets in (X, T).

Example 4. 1: Let X= {a, b, c}. The fuzzy setsλ, μ and ν are defined on X as follows:
λ: X $\rightarrow [0,1]$ is defined as$\lambda(a) = 0.8;\ \lambda(b) = 0.7;\ \lambda(c) = 0.7.$
μ: X $\rightarrow [0,1]$ is definedas $\mu(a) = 0.6;\ \mu(b) = 0.9;\ \mu(c) = 0.8.$
ν: X $\rightarrow [0,1]$ is defined as $\nu(a) = 0.7;\ \nu(b) = 0.5;\ \nu(c) = 0.9.$
Then T $=\{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \mu, \lambda \wedge \nu, \mu \wedge \nu, \lambda \wedge (\mu \vee \nu), \lambda \vee (\mu \wedge \nu), \mu \wedge (\lambda \vee \nu), \mu \vee (\lambda \wedge \nu), \nu \wedge (\lambda \vee \mu), \nu \vee (\lambda \wedge \mu), 1\}$ is a fuzzytopology onX. Now
$\alpha = (1-\lambda) \vee (1 - [\lambda \vee (\mu \wedge \nu)]) \vee [1 - (\lambda \vee \mu \vee \nu)]$and $\text{pint}(\alpha) = 0$
$\beta = (1-\mu) \vee (1 - [\lambda \vee \nu]) \vee [1 - (\mu \vee (\lambda \wedge \nu))]and\text{pint}(\beta) = 0$
$\delta = (1-\nu) \vee (1 - [\lambda \vee \mu]) \vee [1 - (\lambda \wedge \mu)] \vee (1 - [\mu \vee \nu])$and $\text{pint}(\delta) = 0$
$\eta = (1 - [\lambda \wedge \nu]) \vee (1 - [\mu \wedge \nu]) \vee [1 - (\lambda \wedge [\mu \vee \nu]) \vee (1 - [\mu \wedge (\lambda \vee \nu)])$ and $\text{pint}(\eta) = 0$. Thenα, β, δ and η are fuzzy pre σ – nowhere dense sets in (X, T) and also$\text{pint}(\alpha \vee \beta \vee \delta \vee \eta) = 0$and therefore (X, T) is a fuzzy pre σ –Baire space.

Proposition 4. 1: Let (X, T) be a fuzzy topological space. Then the following are equivalent:
(1). (X, T) isa fuzzypre σ –Baire space.
(2). $\text{pint}(\lambda) = 0$, foreach fuzzy pre σ –first category setλin (X, T).
(3). $\text{pcl}(\mu) = 1$, for each fuzzy pre σ –residual set μ in (X, T).

Proof: (1) $\Rightarrow$ (2). Let λ be a fuzzy σ – first category set in (X, T). Then $\lambda = (\vee_{i=1}^{\infty}(\lambda_i))$, where $(\lambda_i)'s$ are fuzzy pre σ – nowhere dense sets in (X, T). Then, $\text{pint}(\lambda) = \text{pint}(\vee_{i=1}^{\infty}(\lambda_i))$. Since (X, T) is a fuzzy pre σ – Baire space, $\text{pint}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$. Hence, $\text{pint}(\lambda) = 0$for a fuzzy pre σ –first category set λ in (X, T).
(2) $\Rightarrow$(3). Letμ be a fuzzy pre σ –residual set μ in (X, T). Then$(1-\mu)$ is a fuzzy pre σ –first category set in (X, T). By hypothesis, $\text{pint}(1-\mu) = 0$. Then, by lemma2. 2, $1 - \text{pcl}(\mu) = 0$. Hence, $\text{pcl}(\mu) = 1$, fora fuzzy pre σ –residual setμ in (X, T).
(3) $\Rightarrow$ (1). Let λ be a fuzzypre σ – first category set in (X, T). Then $\lambda = (\vee_{i=1}^{\infty}(\lambda_i))$, where$(\lambda_i)'$sare fuzzy pre σ –nowhere dense sets in (X, T). Now λ is a fuzzy pre σ –first category set in (X, T), implies that$(1-\lambda)$is a fuzzy pre σ –residual set in (X, T). By hypothesis, $\text{pcl}(1-\lambda) = 1$. Then, by lemma 2. 2, $1 - \text{pint}(\lambda) = 1$ Hence $\text{pint}(\lambda) = 0$. Thatis, $\text{pint}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$ are fuzzy pre σ – nowhere dense sets in (X, T). Hence (X, T) is a fuzzy pre σ –Baire space.

Proposition 4. 2: If $\text{pcl}(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where $(\lambda_i)'$sare fuzzy pre dense and pre G_δ – sets in (X, T), then (X, T) is a fuzzy pre σ –Baire space.

Proof: Now $\mathrm{pcl}\left(\wedge_{i=1}^{\infty}(\lambda_i)\right) = 1$ implies that $1 - \mathrm{pcl}\left(\wedge_{i=1}^{\infty}(\lambda_i)\right) = 0$. Then, by lemma 2. 2, $\mathrm{pint}\left(1 - \wedge_{i=1}^{\infty}(\lambda_i)\right) = 0$ and hence we have $\mathrm{pint}\left(\vee_{i=1}^{\infty}(1-\lambda_i)\right) = 0$. Let $\mu_i = 1 - \lambda_i$. Then, $\mathrm{pint}\left(\vee_{i=1}^{\infty}(\mu_i)\right) = 0$. Since $(\lambda_i)'s$ are fuzzy pre dense and pre $G_\delta -$ sets in (X, T), by proposition 3. 1, $(1-\lambda_i)'s$ are fuzzy pre $\sigma -$ nowhere dense sets in (X, T). Hence, we have$\mathrm{pint}\left(\vee_{i=1}^{\infty}(\mu_i)\right) = 0$, where $(\mu_i)'$sare fuzzy pre $\sigma -$ nowhere dense sets in (X, T). Therefore, (X, T) is a fuzzy pre $\sigma -$Baire space.

Proposition 4. 3: If the fuzzy topological space (X, T) is a fuzzy pre $\sigma -$Baire space, then (X, T) is a fuzzy pre $\sigma -$second category space.

Proof: Let (X, T) be a fuzzy pre $\sigma -$Baire space. Then, $\mathrm{pint}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'$sare fuzzy pre $\sigma -$nowhere dense sets in (X, T). Then, $\vee_{i=1}^{\infty}(\lambda_i) \neq 1_x$ [otherwise, $\vee_{i=1}^{\infty}(\lambda_i) = 1_x$, implies that $\mathrm{pint}(\vee_{i=1}^{\infty}(\lambda_i)) = \mathrm{pint}(1_x) = 1_x$which in turn implies that 0 = 1, a contradiction]. Hence (X, T) is a fuzzy pre $\sigma -$second category space.

Remark 4. 1: The converse of the above proposition need not be true. A fuzzy pre $\sigma -$second category space need not be a fuzzy pre $\sigma -$Baire space. For, considerthe following example:

Example 4. 2: Let X= {a, b, c}. The fuzzy setsλ, μ and $\nu$are defined on X as follows:
$\lambda: X \to [0,1]$is defined as$\lambda(a) = 0.4;\ \lambda(b) = 0.6;\ \lambda(c) = 0.7.$
$\mu: X \to [0,1]$is defined as $\mu(a) = 0.8;\ \mu(b) = 0.4;\ \mu(c) = 0.5.$
$\nu: X \to [0,1]$ is defined as $\nu(a) = 0.7;\ \nu(b) = 0.5;\ \nu(c) = 0.8.$
Then, clearly $T = \{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \mu, \lambda \wedge \nu, \mu \wedge \nu, \lambda \vee (\mu \wedge \nu), \mu \vee (\lambda \wedge \nu), \nu \wedge (\lambda \vee \mu), \lambda \vee \mu \vee \nu, 1\}$ is afuzzy topology on X. Now $\alpha = \{(1-\mu) \vee (1-\nu) \vee (1-\lambda \vee \mu) \vee (1-\lambda \vee \nu) \vee (1-\mu \vee \nu) \vee (1-\lambda \vee \mu \vee \nu)\}$is a fuzzy pre F_σ-set in (X, T) and $\mathrm{pint}(\alpha) = 0$and hence$\alpha$is a fuzzy pre $\sigma -$nowhere dense set in (X, T). $\beta = \{(1-\lambda) \vee (1-[\lambda \wedge \nu]) \vee [1-(\lambda \vee (\mu \wedge \nu)) \vee (1-[\mu \vee (\lambda \wedge \nu)]) \vee (1-\nu \wedge (\lambda \vee \mu)\}$ is a fuzzy pre F_σ-set in (X, T) and $\mathrm{pint}(\beta) = 0$. Hence β is a fuzzy pre $\sigma -$nowhere dense set in (X, T). Now $(\alpha \vee \beta) \neq 1_X$. Therefore (X, T) is a fuzzy pre $\sigma -$second category space. But $p(\alpha \vee \beta) = \lambda \wedge \mu \neq 0$and therefore (X, T) is not a fuzzy pre $\sigma -$Baire space.

Remark 4. 2: A fuzzy pre $\sigma -$Baire space need not be a fuzzy pre Baire space. For, consider the following example:

Example 4. 3: Let X={a, b, c}. The fuzzy setsλ, μ and ν, are defined on X as follows:
$\lambda: X \to [0,1]$ is defined as$\lambda(a) = 1\ ;\ \lambda(b) = 0.2;\ \lambda(c) = 0.7.$
$\mu: X \to [0,1]$is defined as $\mu(a) = 0.3;\ \mu(b) = 1;\ \mu(c) = 0.2.$
$\nu: X \to [0,1]$ is defined as $\nu(a) = 0.7;\ \nu(b) = 0.4;\ \nu(c) = 1.$
Then $T = \{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \mu, \lambda \wedge \nu, \mu \wedge \nu, \mu \vee (\lambda \wedge \nu), \nu \vee (\lambda \wedge \mu), \nu \wedge (\lambda \vee \mu), \lambda \vee \mu \vee \nu, 1\}$is a fuzzy topology on X. Now $(1-\lambda), (1-\mu), (1-\nu), 1-(\lambda \vee \mu), 1-(\lambda \vee \nu), 1-(\mu \vee \nu), 1-(\lambda \wedge \nu), 1-[\mu \vee (\lambda \wedge \nu)], 1-[\nu \vee (\lambda \wedge \mu)], 1-$

$[\nu \wedge (\lambda \vee \mu)]$are fuzzy pre nowhere dense sets in (X, T). $[1-(\lambda \wedge \mu)] = \{(1-\lambda) \vee (1-\mu) \vee (1-\nu) \vee [1-(\lambda \vee \mu)] \vee [1-(\lambda \vee \nu)] \vee [1-(\mu \vee \nu)] \vee [1-(\lambda \wedge \nu)] \vee [1-[\mu \vee (\lambda \wedge \nu)]] \vee [1-[\nu \vee (\lambda \wedge \mu)]] \vee [1-[\nu \wedge (\lambda \vee \mu)]]\}$. Therefore, $1-(\lambda \wedge \mu)$ is a fuzzy pre first category set in (X, T). But $\text{pint}(1-(\lambda \wedge \mu)) = \lambda \wedge \mu \neq 0$. Hence, (X, T) is not a fuzzy preBaire space. Now $\alpha = (1-\nu) \vee (1-[\lambda \vee \nu]) \vee [1-(\mu \vee \nu)]$ and $\text{pint}(\alpha) = 0$. Also $\beta = (1-\lambda) \vee (1-[\lambda \vee \mu]) \vee [1-(\lambda \wedge \nu)] \vee (1-[\mu \vee (\lambda \wedge \nu)] \vee (1-(\mu \vee \nu)]$ and $\text{pint}(\beta) = 0$. Hence α and β is a fuzzy pre $\sigma-$nowhere dense sets in (X, T). Now the fuzzy $(\alpha \vee \beta)$ is a fuzzy pre $\sigma-$first category set in (X, T) and $\text{pint}(\alpha \vee \beta) = 0$. Hence (X, T) is a fuzzy pre $\sigma-$Baire space.

Proposition 4. 4: If the fuzzy topological space (X, T) is a fuzzypre Baire space and if the fuzzy pre nowhere dense sets in (X, T) are fuzzy pre F_σ-sets in(X, T), then (X, T) is a fuzzypre $\sigma-$Baire space.

Proof: Let (X, T) be a fuzzy pre Baire space such that every fuzzy pre nowhere dense set (λ_i) is a fuzzy pre F_σ-set in (X, T). Since (X, T) isa fuzzy pre Baire space, $\text{pint}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$are fuzzy pre nowhere dense sets in (X, T). Also since $(\lambda_i)'s$ are fuzzy pre F_σ-sets andfuzzy pre nowhere dense sets in (X, T), byproposition3. 3, $(\lambda_i)'s$ are fuzzypre $\sigma-$ nowhere densesets in (X, T). Hence, $\text{pint}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$are fuzzy pre $\sigma-$ nowhere dense sets in (X, T), implies that(X, T) is a fuzzy pre $\sigma-$ Baire space.

Proposition 4. 5: Let (X, T) be a fuzzy topological space. If$\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$, where $(\lambda_i)'s$are fuzzy pre denseand fuzzy pre $G_\delta-$ sets in (X, T), then (X, T) is a fuzzy pre $\sigma-$second category space.

Proof: Now$\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$, implies that $1-\wedge_{i=1}^{\infty}(\lambda_i) \neq 1-0 = 1$. Then $\vee_{i=1}^{\infty}(1-\lambda_i) \neq 1$. Since $(\lambda_i)'s$ are fuzzy pre-dense and fuzzy pre $G_\delta-$ set in (X, T), by proposition 3. 1, $(1-\lambda_i)'s$are fuzzy pre $\sigma-$nowhere dense sets in (X, T). Hence, $\vee_{i=1}^{\infty}(1-\lambda_i) \neq 1$, where $(1-\lambda_i)'s$are fuzzy pre $\sigma-$nowhere dense sets in (X, T). Hence (X, T) is not a fuzzy pre $\sigma-$first category space. Therefore (X, T) is a fuzzy pre $\sigma-$second category space.

Proposition4. 6: Ifafuzzy topological space (X, T) is a fuzzy pre $\sigma-$Baire space, thenno non-zero fuzzy pre-open set is a fuzzy$\sigma-$first category set in (X, T).

Proof: Let λbe a non-zero fuzzy pre-open set in a fuzzy pre $\sigma-$Baire space (X, T). Suppose that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where the fuzzy sets $(\lambda_i)'s$ are fuzzy pre $\sigma-$nowhere dense sets in (X, T). Then $\text{pint}(\lambda) = \text{pint}(\vee_{i=1}^{\infty}(\lambda_i))$. Since (X, T) is a fuzzy pre $\sigma-$Baire space, $\text{pint}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$. This implies that $\text{pint}(\lambda) = 0$. Then we will have $\lambda = \text{pint}(\lambda) = 0$, a contradiction, since$\lambda$ is a fuzzy non-zerofuzzy set in (X, T). Hence no non-zerofuzzy pre-open set isa fuzzy pre $\sigma-$first category set in (X, T).

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On Fuzzy Regular σ-Baire Spaces

G. Thangaraj and E. Poongothai

ABSTRACT

In this paper, the concepts of fuzzy regular σ-nowhere dense sets, are introduced and studied. By means of fuzzy regularσ-nowhere dense sets, the concepts of fuzzy regular σ-Baire spaces are introduced and several characterizations of fuzzy regularσ-Baire spaces are studied.

KEYWORDS: Fuzzy dense set, fuzzy G_δ-set, fuzzy F_σ-set, fuzzy σ-nowhere dense set, fuzzy Baire space, fuzzy σ-Baire space, fuzzy sub maximal space, fuzzy Volterra space.

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1. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by **L. A. Zadeh** in his classical paper [12] in the year 1965. This inspired mathematicians to fuzzify mathematical structures. Among the first fields of mathematics to be considered in the context of fuzzy sets was general topology. The concept of fuzzy topology was defined by **C. L. Chang** [3] in the year 1968. The paper of Chang paved the way for the subsequent rapid development along different directions encompassing various concepts and results of general topology for their generalizations in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

The concept of σ–nowhere dense sets was introduced and studied by **Jiling Cao** and **Sina Greenwood** in [4].This concept in fuzzy setting, was introduced and studied by the authors in [9].In this paper, the concepts of fuzzy regular σ-nowhere dense sets, are introduced and studied. By means of fuzzy regular σ-nowhere dense sets, the concepts of fuzzy regular σ-Baire spaces are introduced and several characterizations of fuzzy regularσ-Baire spaces are also studied in this paper.

2.PRELIMINARIES

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang [3]. A fuzzy set λ on a set X is a function from X to [0, 1], that is, λ: X→ [0, 1].

Definition 21: Letλandμ be fuzzy sets in X. Then, for all$x \in X$,

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$

(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$

(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$

(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \psi(x) = \min\{\lambda(x), \mu(x)\}$

(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union$\psi = \vee_i \lambda_i$and intersection $\delta = \wedge_i \lambda_i$, are defined respectively as

(vi) $\psi(x) = sup_i\{\lambda_i(x) / x \in X\}$

(vii) $\delta(x) = inf_i\{\lambda_i(x) / x \in X\}$

Definition 2.2: Let (X, T) be a fuzzy topological space. For a fuzzy set $\lambda$of X, the fuzzy interior and the fuzzy closure, are defined respectively as

(i) int $(\lambda) = \{\mu / \mu \leq \lambda, \mu \in T\}$,

(ii) cl $(\lambda) = \{\mu / \lambda \leq \mu, 1-\mu \in T\}$.

Lemma 2.1 [1]: For a fuzzy set λ of a fuzzy topological space X,

(i). $1 -$ int $(\lambda) =$ cl $(1 - \lambda)$,

(ii). $1 -$ cl $(\lambda) =$ int $(1 - \lambda)$.

Definition 2.3 [6]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, cl $(\lambda) = 1$.

Definition 2.4 [6]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X, T) such that$\mu < cl(\lambda)$.That is, int cl$(\lambda) = 0$.

Definition 2.5[2]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy G_δ-setif $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i \in T$, for each $i \in I$.

Definition 2.6[2]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $1 - \lambda_i \in T$, for each $i \in I$.

Definition 2.7 [9]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy σ −nowhere dense set ifλ is a fuzzyF_σ-set in(X, T) such that int $(\lambda) = 0$.

Definition 2.8 [9]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy σ −first category set if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$ are fuzzyσ-now here dense sets in (X, T). Any other fuzzy set in(X, T) is said to be of fuzzy σ-second category.

Definition 2.9 [1]: For a family $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\vee (cl(\lambda_\alpha)) \leq cl(\vee(\lambda_\alpha))$. In case $\mathcal{A}$ is finite set, $\vee (cl(\lambda_\alpha)) = cl(\vee(\lambda_\alpha))$. Also $\vee (int(\lambda_\alpha)) \leq int(\vee(\lambda_\alpha))$.

Definition 2.10 [8]: Let(X, T) be a fuzzy topological space. Then(X, T) is called a fuzzy Volterra space if $cl\left(\wedge_{i=1}^{N} (\lambda_i)\right) = 1$, where$(\lambda_i)'s$ are fuzzy dense and fuzzy G_δ-sets in (X, T).

Definition 2.11 [10]: A fuzzy topological space (X, T) is called a fuzzy P-space if

countable intersection of fuzzy open sets in (X, T) is fuzzy open. That is, every non-zero fuzzy G_δ-set in (X, T), is fuzzy open in (X, T).

Definition 2.12 [5]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy basically disconnected space if the closure of every fuzzy open F_σ-set is fuzzy open in (X, T).

3. FUZZYREGULARσ-NOWHERE DENSE SETS

Definition 3.1: Let (X, T) be a fuzzy topological space. A fuzzy set λin $(X, T)$is called a fuzzy regular σ −nowhere dense set if $\lambda = \vee_{i=1}^{\infty} \text{cl}(\lambda_i)$, where $\text{int}(\lambda_i) = 0$.

Example 3.1: Let X = {a, b, c}. The fuzzy sets λ, μ and υ are defined on X as follows:

λ: X → [0, 1] is defined as λ (a) = 0.4; λ (b) = 0.5 ; λ (c) = 0.6.

μ: X → [0, 1] is defined as μ (a) = 0.6 ; μ (b) = 0.4 ; μ (c) = 0.5.

υ: X → [0, 1] is defined as υ (a) = 0.7 ; υ (b) = 0.6 ; υ (c) = 0.5.

Then, T = {0, λ, μ, υ, λ ∨ μ, λ ∨υ, μ ∨υ, λ ∧ μ, λ ∧υ, μ∧υ, μ ∨[λ ∧υ], λ ∧[μ ∨υ], υ∧ (λ ∨ μ), λ ∧ μ ∧υ, 1} is clearly a fuzzy topology on X. Now consider the following fuzzy sets defined on X.

α: X → [0, 1] is defined as α (a) = 0.3; α (b) = 0.5 ; α (c) = 0.6.

β: X → [0, 1] is defined as β (a) = 0.4 ; β (b) = 0.3 ; β (c) = 0.5.

δ: X → [0, 1] is defined as δ (a) = 0.6 ; δ (b) = 0.5 ; δ (c) = 0.3.

Now, on computation, int (α) = 0, int (β) = 0, int (δ) = 0 and cl (α) = 1−[υ∧ (λ ∨ μ)], cl (β) = 1 − (μ ∨[λ ∧υ]), cl (δ) = 1−λ and cl (α)∨cl (β)∨cl (δ)=1− (λ ∧υ). Hence 1− (λ ∧υ) is a fuzzy regular σ −nowhere dense set in (X, T).

Proposition 3.1: If λ is a fuzzy regular σ −nowheredense set in a fuzzy topological space (X, T), then $1 - \lambda = \wedge_{i=1}^{\infty} \text{int}(\mu_i)$, where$\text{cl}(\mu_i) = 1$.

Proof: Let λ be a fuzzy regular σ −nowhere dense set in (X, T).Then $\lambda = \vee_{i=1}^{\infty} \text{cl}(\lambda_i)$, where $\text{int}(\lambda_i) = 0$. Now, $\text{int}(\lambda_i) = 0$ implies that $1 - int(\lambda_i) = 1$ and hence we have $\text{cl}(1 - \lambda_i) = 1$. Also $1 - \lambda = 1 - \vee_{i=1}^{\infty} \text{cl}(\lambda_i) = \wedge_{i=1}^{\infty} \text{int}(1 - \lambda_i)$ where $\text{cl}(1 - \lambda_i) = 1$. Let $\mu_i = 1 - \lambda_i$. Then, $1 - \lambda = \wedge_{i=1}^{\infty} \text{int}(\mu_i)$, where $\text{cl}(\mu_i) = 1$.

Proposition 3.2: If $\text{int}(\lambda) = 0$, where λ is a fuzzy regularσ −nowhere dense set in a fuzzy topological space (X, T), thenλis a fuzzy first category set in (X, T).

Proof: Let λbe a fuzzy regular σ −nowhere dense set in a fuzzy topological space (X, T). Then $\lambda = \vee_{i=1}^{\infty} (\text{cl}(\lambda_i))$, where $\text{int}(\lambda_i) = 0$. Now $\text{int}(\lambda) = 0$, implies that $\text{int}\left(\vee_{i=1}^{\infty} (\text{cl}(\lambda_i))\right) = 0$. But $\vee_{i=1}^{\infty} \text{int}(\text{cl}(\lambda_i)) \leq \text{int}\left(\vee_{i=1}^{\infty} \text{cl}(\lambda_i)\right)$ .Hence we have $\vee_{i=1}^{\infty} \text{int}(\text{cl}(\lambda_i)) \leq 0$.That is, $\vee_{i=1}^{\infty} \text{int}(\text{cl}(\lambda_i)) = 0$. This implies that $\text{intcl}(\lambda_i) = 0$. Now $\text{int cl cl}(\lambda_i) = \text{int cl}(\lambda_i) = 0$. Then $(\text{cl}(\lambda_i))'s$ are fuzzy nowhere dense sets in (X, T). Hence $\lambda = \vee_{i=1}^{\infty} (\text{cl}(\lambda_i))$, where $(cl(\lambda_i))'s$ are fuzzy nowhere dense sets, implies that λ is a fuzzy first category set in (X, T).

Proposition3.3: If $\text{int}(\lambda) = 0$, where λ is a fuzzy regular σ −nowhere dense set in a fuzzy topological space (X, T), then $1 - \lambda$ is a fuzzy residual set in (X, T).

Proof: Let λ be a fuzzy regular σ-now here dense set in a fuzzy topological space (X, T) such that $\text{int}(\lambda) = 0$.Then, by proposition 3.2, λ is a fuzzy first category set in

(X, T) and hence $1-\lambda$ is a fuzzy residual set in (X, T).

Remarks: Let λ be a fuzzy regular σ −nowhere dense set in a fuzzy topological space (X, T). Then, we have, $\lambda = \vee_{i=1}^{\infty} \text{cl}(\lambda_i)$, where $\text{int}(\lambda_i) = 0$.If $(\lambda_i)'s$ are fuzzy closed sets in (X, T), then λ is a fuzzy first category set in (X, T).For, if $(\lambda_i)'s$ are fuzzy closed sets, then $\text{cl}(\lambda_i) = \lambda_i$, implies that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $\text{int cl}(\lambda_i) = 0$ and hence λ is a fuzzy first category set in (X, T).Also, since $\text{int}(\lambda_i) = 0$, $(\lambda_i)'s$ are not fuzzy open sets in (X, T).

Proposition 3.4: If λ is a fuzzy regular σ −nowhere dense set in a fuzzy topological space (X, T), then λ is a fuzzyF_σ-set in (X, T).

Proof: Let λ be a fuzzy regular σ −nowhere dense set in a fuzzy topological space (X, T). Then, we have, $\lambda = \vee_{i=1}^{\infty} \text{cl}(\lambda_i)$, where $\text{int}(\lambda_i) = 0$ and hence $\lambda = \vee_{i=1}^{\infty} \text{cl}(\lambda_i)$, implies that λ is a fuzzy F_σ-set in (X, T).

Proposition 3.5: If λ is a fuzzy regular σ −nowhere dense set in a fuzzy topological space (X, T), then $1-\lambda$ is a fuzzy G_δ-set formed from the fuzzy dense sets in (X, T).

Proof: The proof follows from proposition 3.4.

The following proposition gives the condition for a fuzzy regular σ-now here dense set in a fuzzy topological space to be a fuzzy σ −nowhere dense set.

Proposition3.6: If $\text{int}(\lambda) = 0$, for a fuzzy regular σ-now here dense set λ in a fuzzy topological space (X, T), then λ is a fuzzy σ-now here dense set in (X, T).

Proof: Let λ be a fuzzy regular σ-now here dense set in a fuzzy topological space (X, T) such that $\text{int}(\lambda) = 0$. Then, by proposition 3.4, λ is a fuzzy F_σ -set in (X, T). Hence λ is a fuzzy F_σ -set such that $\text{int}(\lambda) = 0$. Therefore λ is a fuzzy σ −nowhere dense set in (X, T).

Theorem 3.1 [10]: If λ is a fuzzy σ −nowhere dense set in a fuzzy P-space(X, T), then λ is not a fuzzy dense set in (X, T).

Proposition 3.7: If $\text{int}(\lambda) = 0$, for a fuzzy regular σ −nowhere dense set λ in a fuzzy P-space (X, T), then λ is not a fuzzy dense set in(X, T).

Proof: Let λ be a fuzzy regular σ −nowhere dense set in a fuzzy topological space (X, T)such that $\text{int}(\lambda) = 0$. Then, by proposition 3.6, $\lambda$is a fuzzyσ −nowhere dense set in (X, T).Since (X, T) is a fuzzy P-space, by theorem 3.1, λ is not a fuzzy dense set in (X, T).

The following proposition ensures the existence of a fuzzy regular σ −nowhere dense set in a fuzzy topological space.

Proposition 3.8: If λ is a fuzzy first category set in a fuzzy topological space(X, T), then there exists a fuzzy regular σ − nowhere dense set μ in (X, T) such that $\lambda \leq \mu$ in (X, T).

Proof: Let λ be a fuzzy first category set in (X, T).Then$\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy nowhere dense setsin$(X, T)$and hence $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, whereint $\text{cl}(\lambda_i) = 0$. Now int $(\lambda_i) \leq \text{int cl}(\lambda_i)$, implies that int $(\lambda_i) = 0$. Let $\mu = \vee_{i=1}^{\infty} \text{cl}(\lambda_i)$, whereint $(\lambda_i) = 0$. Thenμ is afuzzy regular σ − nowhere dense in (X, T). Now $\vee_{i=1}^{\infty}(\lambda_i), \leq \vee_{i=1}^{\infty} \text{cl}(\lambda_i)$, implies that$\lambda \leq \mu$ in (X, T).Therefore, if λ is a fuzzy first category set in a fuzzy topological space(X, T), then there exist a fuzzy regular σ-nowhere dense set μ in (X, T) such that $\lambda \leq \mu$ in (X, T).

Definition3.2. [2]: Afuzzytopologicalspace (X,

T)iscalledafuzzysubmaximalspaceifforeachfuzzyset λin (X, T)such thatcl (λ) = 1, then$\lambda \in$Tin (X, T).

Proposition3.9: If λ is a fuzzy regular σ −nowhere dense set in a fuzzy submaximal space (X, T), then λ is a fuzzy first category set in(X, T).

Proof: Let λ be a fuzzy regular σ −nowhere dense set in a fuzzy submaximal space (X, T). Then, $\lambda = \vee_{i=1}^{\infty}$ cl (λ_i), where int $(\lambda_i) = 0$.Now int $(\lambda_i) = 0$, implies that cl $(1-\lambda_i) = 1-$ int $(\lambda_i) = 1$. Hence $(1-\lambda_i)'s$ are fuzzy dense sets in (X, T). Since (X, T) is a fuzzy submaximal space, the fuzzy dense sets $(1-\lambda_i)'s$ are fuzzy open sets in (X, T) and hence $(\lambda_i)'s$ are fuzzy closed sets in (X, T).Then cl $(\lambda_i) = \lambda_i$. Now int cl cl(λ_i) = int cl(λ_i) = int (λ_i) = 0. Then $(cl(\lambda_i))'$sare fuzzy nowhere dense sets in (X, T). Hence $\lambda = \vee_{i=1}^{\infty} (cl(\lambda_i))$, where $(cl(\lambda_i))'s$ are fuzzy nowhere dense sets, implies that λ is a fuzzy first category set in(X, T).

Theorem 3.2 [7]: Let (X, T) be a fuzzy topological space. Then the following are equivalent:

(1) (X, T) is a fuzzy Baire space.

(2) Int (λ) = 0 for every fuzzy first category set λ in (X, T).

(3) cl (μ) = 1 for every fuzzy residual set μ in (X, T).

Proposition 3.10: If int (λ) = 0, for each fuzzy regular σ −nowhere dense set λ in a fuzzy topological space (X, T), then (X, T) is a fuzzy Baire space.

Proof: Let λ be a fuzzy first category set in (X, T). Then, by proposition 3.8, there exist a fuzzy regular σ −nowhere dense set μ in (X, T) such that $\lambda \leq \mu$ in (X, T). By hypothesis, int (μ) = 0.Now $\lambda \leq \mu$, implies that int $(\lambda) \leq$ int(μ). Then we have$int(\lambda) \leq 0$. Thatis, int (λ) = 0. Hence, by theorem 3.2, (X, T) is a fuzzy Baire space.

Proposition3.11: If λ is a fuzzy regularσ −nowhere dense set in a fuzzyP-space (X, T), thenλ is a fuzzyclosed set in(X, T).

Proof: Let λ be a fuzzy regularσ −nowhere dense set in a fuzzy P-space (X, T). Then, by proposition 3.5, $1-\lambda$ is a fuzzy G_δ-setin (X, T).Since (X, T) is a fuzzy P-space, the fuzzy G_δ-set $(1-\lambda)$is a fuzzy open set in (X, T) and hence λ is a fuzzy closed set in (X, T).

Proposition3.12: If $int[\vee_{i=1}^{N} (\lambda_i)] = 0$, where (λ_i)'s (i= 1 to N) are fuzzy regular σ − nowhere dense sets in a fuzzy topological space (X, T), then (X, T) is afuzzyVolterra space.

Proof: Let (λ_i)'s (i= 1 to N) be fuzzy regular σ-nowhere dense sets in (X, T)such that int $[\vee_{i=1}^{N} (\lambda_i)] = 0$. Now $\vee_{i=1}^{N}[int (\lambda_i]) \leq$ int $[\vee_{i=1}^{N} (\lambda_i)]$, implies that $\vee_{i=1}^{N}$ [int $(\lambda_i)] \leq 0$.That is, $\vee_{i=1}^{N}$ [int $(\lambda_i)] = 0$. This implies that int $(\lambda_i) = 0$, for each i. Then, cl $(1-\lambda_i) = 1-$ int $(\lambda_i) = 1$. Hence $(1-\lambda_i)'s$ are fuzzy dense sets in (X, T). By proposition 3.5, $(1-\lambda_i)'s$ are fuzzy G_δ-sets in (X, T). Thus, $(1-\lambda_i)'s$ are fuzzy dense and fuzzy G_δ -sets in (X, T). Now cl $[\wedge_{i=1}^{N} (1-\lambda_i)]$ = cl $[1-\vee_{i=1}^{N} (\lambda_i)] = 1 - int[\vee_{i=1}^{N} (\lambda_i)] = 1$ and hence (X, T) is afuzzyVolterra space.

Definition 3.3: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy regularσ-first category set if$\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$are fuzzy regular σ-nowhere dense sets in(X, T).

Definition 3.4: Let λ be a fuzzy regular σ-first category set in (X, T). Then 1 – λ is

called a fuzzy regular σ-residual set in (X, T).

Definition 3.5 [11]: A fuzzy topological space (X, T) is called a fuzzy almost resolvable space if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets $(\lambda_i)'s$ in (X, T) are such that int $(\lambda_i) = 0$. Otherwise (X, T) is called a fuzzy almost irresolvable space.

Proposition 3.13: If a fuzzy topological space (X, T) is a fuzzy almost resolvable space, then 1_X is a fuzzy regular σ-nowhere dense set.

Proof: Let (X, T) be a fuzzyalmost resolvable space. Then, $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets $(\lambda_i)'s$ in (X, T) are such thatint $(\lambda_i) = 0$.Now $\vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty} cl(\lambda_i)$, implies that $1 \leq \vee_{i=1}^{\infty} cl(\lambda_i)$. That is, $\vee_{i=1}^{\infty} cl(\lambda_i) = 1$, where int $(\lambda_i) = 0$.Hence 1 is afuzzy regular σ-nowhere dense set.

Proposition 3.14: If λ is a fuzzy open and fuzzy regular σ-nowhere dense set in a fuzzy topological space (X, T), then $1-\lambda$ isafuzzy closed andfuzzy G_δ-setin (X, T).

Proof: Let λ be a fuzzy open and fuzzy regular σ −nowhere dense set in (X, T). Since λ is a fuzzy regular σ −nowhere dense set, by proposition 3.5, $1-\lambda$ is a fuzzy G_δ-set in (X, T). Hence $1-\lambda$ is a fuzzy closed and fuzzy G_δ-setin (X, T).

Theorem 3.3[5]: For any fuzzy topological space (X, T), the following are equivalent:

(a) (X, T) is fuzzy basically disconnected.

(b) For each fuzzy closed G_δ-set λ, int (λ) is fuzzy closed.

(c) For each fuzzy open F_σ-set λ, cl (λ) + cl $(1 - cl(\lambda)) = 1$.

Proposition 3.15: If a fuzzy regular σ −nowhere dense set λ is a fuzzy open set in a fuzzy basically disconnected space (X, T), then cl (λ) is a fuzzy open set.

Proof: Let the fuzzy regular σ −nowhere dense set λ be a fuzzy open set in (X, T). Then, by proposition 3.14, $1-\lambda$ is a fuzzyclosed and fuzzy G_δ-set in (X, T). Since (X, T)is a fuzzy basically disconnected space, by theorem 3.3, int $(1-\lambda)$ is a fuzzy closed set in (X, T). That is, cl [int $(1-\lambda)$] =int $(1-\lambda)$ and hence $1-$int cl $(\lambda) = 1-$ cl (λ). This implies that, cl (λ) = int cl (λ). Therefore cl (λ) is a fuzzy open set in (X, T).

Proposition 3.15: If int $(\lambda) = 0$, for a fuzzy regular σ-nowhere dense set λ in a fuzzy P-space (X, T), then λ is a fuzzynowhere dense set in (X, T).

Proof: Let λ be a fuzzy open and fuzzy regular σ-nowhere dense set in a fuzzy P-space (X, T) such that int $(\lambda) = 0$.Then, by proposition 3.11, λ is a fuzzyclosed set in (X, T) and hence cl $(\lambda) = \lambda$. Now int cl $(\lambda) = int(\lambda) = 0$ and therefore λ is a fuzzynowhere dense set in (X, T).

4. FUZZY REGULAR σ-BAIRE SPACES

Definition 4.1: A fuzzy topological space (X, T) is called a fuzzy regular σ-Baire space if $\text{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy regularσ-nowhere dense sets in (X, T).

The following proposition gives the characterization of fuzzy regularσ-Baire space in terms offuzzy regular σ-first category sets and fuzzy regular σ-residual sets in a fuzzy topological space.

Proposition 4.1: Let (X, T) be a fuzzy topological space. Then the following are equivalent:

(1) (X, T) is a fuzzy regular σ-Baire space.
(2) Int (λ) = 0 for every fuzzy regular σ-first category set λ in (X, T).
(3) cl (μ) = 1 for every fuzzy regular σ-residual set μ in (X, T).

Proof: **(1) $\Rightarrow$ (2).** Let λ be a fuzzy regular σ-first category set in (X, T). Then $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy regular σ-nowhere dense sets in (X, T) and hence int (λ) = int $(\bigvee_{i=1}^{\infty} (\lambda_i))$.Since (X, T) is a fuzzy regular σ-Baire space, int $(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$. Hence int ($\lambda$) = 0 for a fuzzy regular σ-first category set λin (X, T).

(2) $\Rightarrow$ (3). Let μ be a fuzzy regular σ-residual set in (X, T). Then $(1 - \mu)$ is a fuzzy regular σ-first category set in (X, T). Byhypothesis, int $(1 - \mu) = 0$. Then $1 - cl(\mu) = int(1 - \mu) = 0$.Hence cl ($\mu$)= 1, for a fuzzy regular σ-residual setμ in (X, T).

(3) $\Rightarrow$ (1). Let λ be a fuzzy regular σ-first category set in (X, T). Then $\lambda = (\bigvee_{i=1}^{\infty} (\lambda_i))$, where (λ_i)'s are fuzzy regularσ-nowhere dense sets in (X, T). Now λ is afuzzyregular σ-first category set in (X, T), implies that $(1 - \lambda)$ is a fuzzy regular σ-residual set in (X, T). By hypothesis, cl $(1 - \lambda) = 1$. Then $1 - int(\lambda) = 1$ and hence int (λ) = 0. That is, int $(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$, where $(\lambda_i)'s$ are fuzzy regular σ-nowhere dense sets in (X, T). Hence (X, T) is a fuzzy σ-Baire space.

Proposition 4.2: If a fuzzy topological space (X, T) is a fuzzy regularσ-Baire space, then, cl $(\bigwedge_{i=1}^{\infty} (\mu_i)) = 1$, where $(\mu_i)'s$ are fuzzy G_δ-sets formed from the fuzzy dense setsin (X, T).

Proof: Let (X, T) be a fuzzy regular σ-Bairespace. Then, int $(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i)'s are fuzzy regular σ –nowheredense setsin (X, T). Now $1 - int(\bigvee_{i=1}^{\infty} (\lambda_i)) = 1$, implies that cl $(\bigwedge_{i=1}^{\infty} (1 - \lambda_i)) = 1$. Since (λ_i)'s are fuzzy regularσ –nowheredense setsin (X, T), by proposition 3.1, we have $1 - \lambda_i = \wedge_{j=1}^{\infty} int(\mu_{ij})$, where$cl(\mu_{ij}) = 1$. Let $\delta_i = 1 - \lambda_i$. Then, $(\delta_i)'s$ are fuzzy G_δ -sets in (X, T). Hence we have cl $(\bigwedge_{i=1}^{\infty} (\delta_i)) = 1$, where $(\delta_i)'s$ are fuzzy G_δ-sets formed from the fuzzy dense sets in (X, T).

Proposition 4.3: If a fuzzy topological space (X, T) is a fuzzy regular σ $-$Baire space, then, (X, T) is a fuzzy Baire space.

Proof: Let (X, T) bea fuzzyregular σ-Bairespace. Then, int $(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i)'s are fuzzy regular σ-nowheredense sets in (X, T).Now $\bigvee_{i=1}^{\infty}[int(\lambda_i]) \leq int[\bigvee_{i=1}^{\infty} (\lambda_i)]$, implies that $\bigvee_{i=1}^{\infty} [int(\lambda_i)] \leq 0$. That is, $\bigvee_{i=1}^{\infty} [int(\lambda_i)] = 0$. This implies that int $(\lambda_i) = 0$, for each i. Thus, for the fuzzy regular σ-nowheredense sets $(\lambda_i)'s$, in (X, T), we have int $(\lambda_i) = 0$. Therefore by proposition 3.10**,** (X, T) is a fuzzy Baire space.

Proposition 4.4: If (X, T) is a fuzzy regular σ-Baire space, then $int(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i)'s are fuzzy first category sets in(X, T).

Proof: Let (λ_i)'s be fuzzy first category sets in(X, T).Then, by proposition 3.8, there exist fuzzy regular σ-nowhere dense sets μ_i in (X, T) such that $\lambda_i \leq \mu_i$ in (X, T). Then we have $int(\bigvee_{i=1}^{\infty} (\lambda_i)) \leq int(\bigvee_{i=1}^{\infty} (\mu_i))$. Since $(X, T)$is a fuzzy regularσ-Baire space, $int(\bigvee_{i=1}^{\infty} (\mu_i)) = 0$. Then, $int(\bigvee_{i=1}^{\infty} (\lambda_i)) \leq 0$. That is, $int(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$. Thus, $int(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i)'s are fuzzy first category sets in(X, T).

Proposition 4.5: If(X, T) is a fuzzy regular σ-Baire space, then (X, T) is a fuzzy Baire

space.

Proof: Let λ be a fuzzy first category set in (X, T). Then, $\lambda \leq \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy first category sets in (X, T) in which $\lambda_1 = \lambda$.Now, $\lambda \leq \vee_{i=1}^{\infty} (\lambda_i)$impliesthat int $(\lambda) \leq$ int $(\vee_{i=1}^{\infty} (\lambda_i))$.Since (X, T) is a fuzzy regular σ-Baire space, by proposition 4.4, $\text{int}(\vee_{i=1}^{\infty} (\lambda_i)) = 0$.Then, int $(\lambda) \leq 0$. That is, int $(\lambda) = 0$. Hence we have int $(\lambda) = 0$, for a fuzzy first category set in (X, T) and therefore, by theorem 3.2, (X, T) is a fuzzy Baire space.

Definition 4.2 [8]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Volterra space if $\text{cl}\left(\wedge_{i=1}^{N} (\lambda_i)\right) = 1$, where $(\lambda_i)'s$ are fuzzy dense and fuzzy G_δ-sets in (X, T).

Definition 4.3 [9]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy σ-Baire Space if int $(\vee_{i=1}^{\infty} (\lambda_i)) = 0$, where $(\lambda_i')s$ are fuzzyσ-nowhere dense sets in (X, T).

Proposition4.6: If (X, T) is a fuzzy regular σ -Baire space, then (X, T) is afuzzyVolterra space.

Proof: Let (X, T) be afuzzy regular σ-Baire space.Then, $\text{int}(\vee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i) 's are fuzzy regular σ -nowhere dense sets in (X, T). Now $\vee_{i=1}^{\infty}[\text{int} (\lambda_i]) \leq \text{int} [\vee_{i=1}^{\infty} (\lambda_i)]$, implies that $\vee_{i=1}^{\infty} [\text{int} (\lambda_i)] \leq 0$. That is, $\vee_{i=1}^{\infty} [\text{int} (\lambda_i)] = 0$. This implies that $\text{int} (\lambda_i) = 0$, for each i. Thus, for the fuzzy regular σ-nowheredense sets $(\lambda_i)'s$ in (X, T), we have $\text{int} (\lambda_i) = 0$. Then cl $(1 - \lambda_i) = 1 -$ int $(\lambda_i) = 1$.Hence $(1 - \lambda_i)'s$ are fuzzy dense sets in (X, T). By theorem 3.5, $(1 - \lambda_i)'s$ are fuzzy G_δ-sets in (X, T). Let $\mu_i = 1 - \lambda_i$, (i = 1 to N). Then, $\wedge_{i=1}^{\infty} (1 - \lambda_i) \leq \wedge_{i=1}^{N} (\mu_i)$ and hence cl$[\wedge_{i=1}^{\infty} (1 - \lambda_i)] \leq \text{cl} [\wedge_{i=1}^{N} (\mu_i)]$.This implies that $1 - \text{int}(\vee_{i=1}^{\infty} (\lambda_i)) \leq \text{cl} [\wedge_{i=1}^{N} (\mu_i)]$ and hence $1 - 0 \leq \text{cl} [\wedge_{i=1}^{N} (\mu_i)]$. That is, $\text{cl} [\wedge_{i=1}^{N} (\mu_i)] = 1$ and therefore (X, T) is afuzzy Volterra space.

Proposition4.7: If (X, T) is a fuzzy regular σ-Baire space, then (X, T) is a fuzzy σ-Baire space.

Proof: Let (X, T) be a fuzzyregular σ-Bairespace. Then, int $(\vee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i)'s are fuzzy regular σ –nowheredense sets in (X, T).Now $\vee_{i=1}^{\infty}[\text{int} (\lambda_i]) \leq \text{int} [\vee_{i=1}^{\infty} (\lambda_i)]$, implies that$\vee_{i=1}^{\infty} [\text{int} (\lambda_i)] \leq 0$. That is, $\vee_{i=1}^{\infty} [\text{int} (\lambda_i)] = 0$. This implies that $\text{int} (\lambda_i) = 0$, for each i. By proposition 3.6, (λ_i)'s arefuzzy σ-nowheredense sets in (X, T).Hence we have, int $(\vee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i)'s are fuzzy σ-nowheredense sets in (X, T).Therefore (X, T) is a fuzzy σ-Baire space.

Definition 4.4: A fuzzy topological space (X, T) is called a fuzzy regular σ-first category space if the fuzzy set $\mathbf{1_X}$ is a fuzzy regular σ-first category set in (X, T). That is, $1_X = \vee_{i=1}^{\infty} (\lambda_i)$ where $(\lambda_i)'s$ are fuzzy regular σ-nowhere dense sets in (X, T). Otherwise, (X, T) will be called a fuzzy regular σ-second category space.

Proposition 4.8: If a fuzzy topological space (X, T) is a fuzzy regular σ-first category space, then$\wedge_{i=1}^{\infty} (\mu_i) = 0$, where $(\mu_i)'s$ are fuzzy G_δ-sets formed from the fuzzy dense sets in (X, T).

Proof: Let (X, T) be a fuzzy regular σ-first category space. Then, $1 = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$ are fuzzy regular σ-nowhere dense sets in (X, T).Now $1 - \vee_{i=1}^{\infty} (\lambda_i) = 0$.Then, $\wedge_{i=1}^{\infty} (1 - \lambda_i)) = 0$. Since (λ_i)'s are fuzzy regularσ –nowheredense setsin (X,

T), by proposition 3.5, $(1-\lambda_i)'s$ are fuzzy G_δ-sets formed from the fuzzy dense sets in (X, T).Let $\mu_i = (1-\lambda_i)$. Then $\wedge_{i=1}^{\infty}(\mu_i) = 0$, where $(\mu_i)'s$ are fuzzy G_δ-sets formed from the fuzzy dense sets in (X, T).

Proposition 4.9: If a fuzzy topological space (X,T) is a fuzzy regular σ −Baire space, then (X, T) is a fuzzy regular σ-second category space.

Proof: Let (X, T) be a fuzzyregularσ-Baire space. Thenint $(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where (λ_i)'s are fuzzy regular σ-nowhere dense sets in (X, T). Now we claim that $\vee_{i=1}^{\infty}(\lambda_i) \neq 1_x$.Suppose that $\vee_{i=1}^{\infty}(\lambda_i) = 1_x$.Then, this will imply that int $(\vee_{i=1}^{\infty}(\lambda_i))$ = int $(1_x)=1_x \neq 0$, a contradiction. Hence we must have $\vee_{i=1}^{\infty}(\lambda_i) \neq 1_x$ and (X, T) isnot a fuzzy regular σ-first category space. Therefore (X, T) is a fuzzy regular σ-second category space.

Proposition 4.10: If the fuzzy topological space (X,T) is a fuzzy regular σ-first category space, then$(X,T)$is not a fuzzy regularσ-Baire space.

Proof: Let the fuzzy topological space (X,T)be a fuzzy regularσ-first category space. Then, $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets$(\lambda_i)'$s are fuzzy regular$\sigma$-nowhere dense sets in (X,T).Now, $\text{int}\left(\vee_{i=1}^{\infty}(\lambda_i)\right) = \text{int}(1) = 1 \neq 0$. Hence (X,T) is not a fuzzy regularσ-Baire space.

Proposition 4.11: If thefuzzy topological space(X,T) is a fuzzy regularσ-Baire space, then no non-zero fuzzy open set is a fuzzy regularσ-first category set in (X,T).

Proof: Let λbe a non-zero fuzzyopen set ina fuzzyregularσ-Baire space(X,T).such that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where thefuzzy sets$(\lambda_i)'$s are fuzzy regular σ-nowhere dense sets in (X,T). Then $\text{int}(\lambda) = \text{int}\left(\vee_{i=1}^{\infty}(\lambda_i)\right)$. Since (X,T) is a fuzzy regular σ-Baire space, $\text{int}\left(\vee_{i=1}^{\infty}(\lambda_i)\right) = 0$. This implies that $\text{int}(\lambda) = 0$. Then we will have $\lambda = 0$, a contradiction, [since$\lambda \in T, \text{int}(\lambda) = \lambda$ in (X,T)]. Hence $\lambda \neq \vee_{i=1}^{\infty}(\lambda_i)$and therefore no non-zero fuzzy open set is a fuzzy regular σ-first category set in (X,T).

Proposition 4.12: If thefuzzy topological space(X,T) is a fuzzy regularσ-Baire space, then $(X,T)$is a fuzzy almost irresolvable space.

Proof: Let (X,T)be afuzzy regularσ-Baire space. By proposition 4.9, $(X,T)$is a fuzzy regular σ-second category space and hence (X,T) is not a fuzzy regular σ-first categoryspace. Then, $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where $(\lambda_i)'$ sare fuzzy regular σ -nowhere densesets in (X,T). Now, $\vee_{i=1}^{\infty}[\text{int}(\lambda_i]) \leq \text{int}[\vee_{i=1}^{\infty}(\lambda_i)\ ]$, implies that $\vee_{i=1}^{\infty}[\text{int}(\lambda_i)] \leq 0$. That is, $\vee_{i=1}^{\infty}[\text{int}(\lambda_i)] = 0$. This implies that $\text{int}(\lambda_i) = 0$, for each i.

Hence $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where $\text{int}(\lambda_i) = 0$ andtherefore (X,T) is a fuzzy almost irresolvable space.

REMARKS

The relation betweenfuzzy regularσ-Baire space, fuzz σ-Baire spaceand fuzzy almost irresolvable space, can be summarized as follows:

Fuzzy regularσ-Baire space,

Fuzzyσ-Baire Space

Fuzzy Almost Irresolvable Space

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On Fuzzy Soft Baire spaces

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ABSTRACT-- Some new concepts of Baireness in fuzzy soft topological spaces are introduced, and their characterizations and properties are investigated in this work. In continuation of earlier work fuzzy soft nowhere dense set we further investigate several properties and characterizations of fuzzy soft Baire spaces.

Keywords: Fuzzy soft dense, fuzzy soft nowhere dense, fuzzy soft first category, fuzzy soft second category, fuzzy soft G_δ- set, fuzzy soft Baire space.

1.Introduction

The concept of Baire spaces have been studied extensively in classical topology in [1]. In 2013 the concept of baire space in fuzzy setting was introduced and studied G.Thangaraj and s.Anjalmose[5]. Later other authors like Maji et al.[3-4] have further studied the theory of soft sets and used this theory to solve some decision making problems.

Next, the concept of fuzzy soft set is introduced and studied [2] a more generalized concept, which is a combination of fuzzy set and soft set and studied its properties. Since then much attention has been paid to generalize the basic concepts of fuzzy topology in soft setting and this modern theory of fuzzy soft topology has been developed. In recent years, fuzzy soft topology has been found to be very useful in solving any practical problems. The aim of this paper is to introduce the concepts of fuzzy soft Baireness in fuzzy soft topological spaces. In this paper we discuss several characterizations of fuzzy soft dense, fuzzy soft G_δ- set, fuzzy soft nowhere dense, fuzzy soft first category, fuzzy soft second category, fuzzy soft Baire spaces.

2.preliminaries

Definition 2.1:

The fuzzy soft set $F_\phi \in FS(U, E)$ is said to be null fuzzy soft set and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fuzzy soft set $\bar{0}$ of U, where $\bar{0}(x) = 0$ for all $x \in U$.

Definition 2.2:

Let $FE \in FS(U, E)$ and $FE(e) = \bar{1}$ all $e \in E$, where $\bar{1}(x) = 1$ for all $x \in U$. Then F_E is called absolute fuzzy soft set. It is denoted by $\bar{E}$.

Definition 2.3:

A fuzzy soft set FA is said to be a fuzzy soft subset of a fuzzy soft set GB over a common universe U if $A \subseteq B$ and $F(e) \subseteq GB(e)$ for all $e \in A$, *i.e.*, if $\mu_{F(e)}(x) \leq \mu_{G(e)}(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F \tilde{\subseteq} GB$.

Definition 2.4:

Two fuzzy soft sets F and G_B over a common universe U are said to be fuzzy soft equal if FA is a fuzzy soft subset of G and G_B is a fuzzy soft subset of F.

Definition 2.5:

The union of two fuzzy soft sets FA and GB over the common universe U is the fuzzy soft set HC, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \tilde{\vee} G_B$.

Definition 2.6:

Let F_A and G_B be two fuzzy soft set, then the intersection of F_A and G_B is a fuzzy soft set H_C, defined by $H(e) = \mu^e_{F_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \tilde{\wedge} G_B$.

Definition 2.7:

Let $F_A \in FS(U, E)$ be a fuzzy soft set. Then the complement of F_A, denoted by F_A^C, defined by

$$F_A^C(e) = \begin{cases} \bar{1} - \mu^e_F A, & if\ e \in A \\ \bar{1}, & if\ e \notin A \end{cases}$$

Definition 2.8:

Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:

(i) $\phi, \bar{E}$ belong to ψ.

(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ.

(iii) The intersection of any two fuzzy soft sets ψ belongs to ψ. The triplet (U, E, ψ) is called a fuzzy soft topological space over U. The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

Definition 2.9:

The union of all fuzzy soft open subsets of F_A over (U, E, ψ) is called the interior of F_A and is denoted by $int^{fs}(F_A)$.

Definition 2.10:

The union of all fuzzy soft open subsets of F_A over (U, E) is called the interior of F_A and is denoted by $int^{fs}(F_A)$.

Definition 2.11:

Let $F_A \in FS(U,E)$ be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.

3. FUZZY SOFT BAIRE SPACES

Definition 3.1:

Let (U,E,ψ) be a fuzzy soft topological space. Then (U,E,ψ) is called a fuzzy soft baire space if $int^{fs}(\vee_{i=1}^{\infty}(F_{A_i}))=0$ where F s are fuzzy soft nowhere dense sets in (U,E,ψ).

Proposition3.2:

If F_A is a fuzzy soft dense and fuzzy G -set in a fuzzy soft topological space (U, E, ψ), then 1−F is a fuzzy soft first category set in (U, E, ψ).

Proof:

Since F_A is a fuzzy soft G set in (U, E, ψ), $F_A=\wedge_{i=1}^{\infty}(F_{A_i})$ where $F_A \in T$ and since F is a fuzzy soft dense in (U, E, ψ), $cl^{fs}(F_A)=1$, then $cl^{fs}(\wedge_{i=1}^{\infty}(F_{A_i}))=1$. But $cl^{fs}(\wedge_{i=1}^{\infty}(F_{A_i}))\leq\wedge_{i=1}^{\infty} cl^{fs}(F_{A_i})$. Hence $1\leq\wedge_{i=1}^{\infty} cl^{fs}(F_{A_i})$. (ie) $\wedge_{i=1}^{\infty} cl^{fs}(F_{A_i})=1$. Then we have $cl^{fs}(F_{A_i})=1$ for each $F_{Ai}\in T$ and hence $cl^{fs}(int\, F_{Ai})=1$. which Implies that $1- cl^{fs}\, int^{fs}(F_{A_i})=0$ and hence $int^{fs}\, cl^{fs}(1-F_{A_i})=0$. Therefore $1-F_{A_i}$ is a fuzzy soft nowhere dense set in (U,E,ψ). Now $1-F_{A_i}=1-\wedge_{i=1}^{\infty}(F_{A_i}) = \vee_{i=1}^{\infty}(1-F_{A_i})$ therefore $1-F_{A_i}=\vee_{i=1}^{\infty}(1-F_{A_i})$ where $(1-F_{A_i})$'s are fuzzy soft nowhere dense sets in (U, E, ψ). Hence $1-F_{A_i}$ is a fuzzy soft first category set in (U, E, ψ).

Definition 3.3:

Let F_A be a fuzzy soft first category set in a fuzzy soft topological space (U,E,ψ). Then $1-F_A$ is called a fuzzy soft residual set in (U,E,ψ).

Proposition 3.4:

If F_A is a fuzzy soft dense and fuzzy soft G_δ set in a fuzzy soft topological space (U, E, ψ), then F_A is a fuzzy soft residual set in (U,E,ψ).

Proof:

Since F_A is a fuzzy soft dense and fuzzy soft G_δ set in (U, E, ψ), by proposition 3.2, we have $1-F_A$ is a fuzzy soft first category set in (U,E,ψ) and hence F_A is a fuzzy soft residual set in (U, E, ψ).

Proposition 3.5:

If the fuzzy soft topological space (U, E, ψ) has a fuzzy soft dense and fuzzy soft G_δ set, then (U, E, ψ) is a fuzzy soft Baire space.

Proof:

Let F_A be a fuzzy soft dense and G_δ set in (U, E,ψ). Then by proposition 3.2, $1-F_A$ is a fuzzy soft first category set in (U,E,ψ) and $(1-F_A) = \vee_{i=1}^{\infty}(1-F_{A_i})$ where $(1-FA_i)$'s are fuzzy soft nowhere dense sets in (U, E, ψ). But $int^{fs}(1-F_A)=1- cl^{fs}(F_A)=1-1=0$ (since F_A is fuzzy soft dense, $cl^{fs}(F_A)=1$). Then $int^{fs}(\vee_{i=1}^{\infty}(1-F_{A_i}))= int^{fs}(1-F_A) = 0$ and hence (U,E,ψ) is a fuzzy soft baire space.

Proposition 3.6:

If the fuzzy soft topological space (U,E,ψ) is a fuzzy soft first category space, then (U,E,ψ) is not a fuzzy soft baire space.

Proof:

Since (U, E, ψ) is a fuzzy soft first category space, then $\vee_i^{\infty} F = 1$, where F_{A_i}'s are fuzzy soft nowhere dense sets in (U, E, ψ). Therefore $int(\vee_i^{\infty}(F)) = int(1) = 1 \neq 0$. Hence $int(\vee_i^{\infty}(F)) \neq 0$, where F_{A_i}'s are fuzzy soft nowhere dense sets in (U, E, ψ) and therefore (U, E, ψ) is not a fuzzy soft Baire space.

Definition 3.7:

Let (U, E, ψ) be a fuzzy soft topological space. Then (U, E, ψ) is called a fuzzy soft baire space if $int^{fs}(\vee_{i=1}^{\infty}(F_{A_i}))=0$, where F_{A_i}'s are fuzzy soft nowhere dense sets in (U, E, ψ).

EXAMPLE 3.8:

Let $U=\{C_1,C_2,C_3\}$ be the set of three flats and $E=\{e_1,e_2,e_3\}$, where e_1,e_2,e_3 stand for costly, modern, security services. Then we consider

$$F_E=\begin{bmatrix}.6 & .5 & .7\\ .6 & .4 & .6\\ .5 & .3 & .4\end{bmatrix},$$

$$G_E=\begin{bmatrix}.6 & .8 & .7\\ .4 & .7 & .7\\ .3 & .6 & .5\end{bmatrix} H_E=\begin{bmatrix}.6 & .5 & .8\\ .5 & .3 & .7\\ .4 & .2 & .6\end{bmatrix}$$

$T^1_E=F_E\wedge G_E$, $T^2_E=F_E\wedge H_E$, $T^3_E=G_E\wedge H_E$, $T^4_E=F_E\vee G_E$, $T^5_E=F_E\vee H_E$, $T^6_E=G_E\vee H_E$, $T^7_E=F_E\wedge(G_E\vee H_E)$, $T^8_E=F_E\vee(G_E\wedge H_E)$, $T^9_E=G_E\wedge(F_E\vee H_E)$

$T^{10}_E=G_E\vee(F_E\wedge H_E)$, $T^{11}_E=H_E\wedge(F_E\vee H_E)$, $T^{12}_E=H_E\vee(F_E\wedge G_E)$, $T^{13}_E=F_E\vee G_E\vee H_E$ and $T^{14}_E=F_E\wedge G_E\wedge H_E$.

Now consider the fuzzy soft sets $\alpha = (1-F_E)\vee(1-T^8_E)\vee(1-T^{13}_E)=\begin{bmatrix}.6 & .5 & .7\\ .6 & .6 & .7\\ .5 & .7 & .6\end{bmatrix}$,

$\beta = (1-G_E) \vee (1-T^5_E) \vee (1- T^{10}_E) = \begin{bmatrix} .4 & .5 & .3 \\ .6 & .6 & .3 \\ .7 & .7 & .5 \end{bmatrix}$ and $\upsilon = (1- H_E) \vee (1- T^1_E) \vee (1- T^4_E) \vee (1- T^6_E) = \begin{bmatrix} .4 & .5 & .3 \\ .6 & .7 & .4 \\ .7 & .8 & .6 \end{bmatrix}$ in (U, E, ψ). Then α, β, and υ are fuzzy soft sets in (U, E, ψ) and $int^{fs}(\alpha)=0$, $int^{fs}(\beta) = 0$ and , $int^{fs}(\upsilon)=0$. Then α, β, υ are fuzzy soft nowhere dense sets in (U, E,ψ).

Moreover, $(1-T^2_E) \vee (1- T^3_E) \vee (1- T^7_E) \vee (1- T^9_E) \vee (1- T^{11}_E) \vee (1- T^{12}_E) \vee (1- T^{14}_E) = \upsilon$ and also $int^{fs}(\alpha \vee \beta \vee \upsilon) = int^{fs}(\upsilon)=0$ and therefore (U,E,ψ) is a fuzzy soft baire space.

Proposition 3.9:

Let (U, E,ψ) be a fuzzy soft topological space. Then the following are equivalent:

(1)(U, E, ψ) is a fuzzy soft Baire space.

(2)$int^{fs}(F_A)=0$ For every fuzzy soft first category

Set in F_A in (U, E,ψ).

(3) $cl^{fs}(G_B)= 1$ for every fuzzy soft residual set G_B In (U,E, ψ).

Proof:

(**1**)$\Longrightarrow$(**2**). Let $F_A$be a fuzzy soft first category set in (U, E, ψ).Then $F_A=\bigvee_i^{\infty}(F)$,where F s are fuzzy soft nowhere dense sets in (U,E,ψ).Then, we have $int^{fs}(F_A) = int(\bigvee_i^{\infty}(F))$. Since (U, E, ψ) is a fuzzy soft baire space, $(int \bigvee_i^{\infty}(F))=0$. Hence $int^{fs}(F_A)=0$ for any fuzzy soft first category set F_A in (U, E, ψ).

(**2**)$\Longrightarrow$(**3**). Let G_B be a fuzzy soft residual set in (U, E,ψ). Then 1-G_B is a fuzzy soft first category set in (U, E,ψ). By hypothesis, int (1-G) =0. Then 1- $cl^{fs}(G_B)=0$. Hence $cl^{fs}(G_B)= 1$ for any fuzzy soft residual set G_B in (U,E, ψ).

(**3**)$\Longrightarrow$(**1**). Let F_A be a fuzzy soft first category set in (U, E, ψ).$F_A=\bigvee_{i=1}^{\infty}(F_{A_i})$,where F_{A_i} s are fuzzy soft nowhere dense sets in (U,E,ψ). Now F_A is a fuzzy soft first category set in (U, E,ψ) implies that (1-F_A) is a fuzzy soft residual set in (U, E, ψ). By hypothesis, we have $cl^{fs}(1-F_A)=1$. Then 1- $int^{fs}(F_A)=1$.Hence $int^{fs}(F_A)=0$. That is, $int^{fs}(\bigvee_{i=1}^{\infty}(F_{A_i}) =0$ where F_{A_i} s are fuzzy soft nowhere dense sets in (U, E, ψ). Hence (U, E, ψ) is a fuzzy soft baire space.

Definition 3.10:

A fuzzy soft topological space (U, E, ψ) is called fuzzy soft first category space if the fuzzy soft set 1_x is a fuzzy soft first category set in (U, E, ψ). That is ,$1_x = \bigvee_{i=1}^{\infty}(F_{A_i})$ where F_{A_i}s are fuzzy soft nowhere dense sets in (U, E, ψ). Otherwise (U, E, ψ) will be called a fuzzy soft second category space.

Proposition3.11:

If the fuzzy topological space (U, E, ψ) is a fuzzy soft first category space, then (U, E, ψ) is not a fuzzy soft Baire space .

Proof :

Let the fuzzy soft topological space (U, E, ψ) is a fuzzy soft first category space. Then $\bigvee_{i=1}^{\infty} F_{A_i}=1$,Where F_{A_i}sare fuzzy soft nowhere dense sets in (U,E,ψ). Now $int^{fs}(\bigvee_{i=1}^{\infty}(F_{A_i}) = int^{fs}(1)\neq 0$. Hence by definition, (U, E, ψ) is not a fuzzy soft Baire space.

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On Fuzzy soft Regularly nowhere dense sets

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Abstract. In this paper, several characterizations of fuzzy soft regularly nowhere dense sets, fuzzy soft regularly dense, fuzzy soft regularly residual, several examples are given to illustrate the concepts introduced in this paper.

Keywords: Fuzzy soft regularly dense; fuzzy soft regularly nowhere dense set ; fuzzy soft regularly first category ; fuzzy soft regularly residual .

Introduction

Modeling uncertain data has been an interesting subject in engineering, eco- nomics, environmental and social sciences. Crisp set theory helped a little for formal modeling, reasoning and computing uncertain data. However, there are many com- plex problems in various branches of science that involve data which is not always crisp. We can not successfully use classical methods because of various types of uncertainties present in these problems.In 1999 Molodtsov[8] introduced the Soft set theory (SST) as a new mathematical tool to deal with uncertain data which is free from such difficulties. This theory has proved to be useful in many different fields such as:decision making ,data analysis forecasting and simulation .Later on Maji et al. [7] introduced several operations of soft sets. Pei and Miao [10], M. I. Ali et al. [1] introduced and studied several soft set operations. The concept of FSS is introduced and studied [3-6, 9] a more generalized concept, which is a combination of fuzzy set and fuzzy soft set and studied its properties. The aim of this paper is to introduce the concepts of fuzzy soft regularly nowhere dense sets.

2.Preliminaries

2.1[3].Definition

The fuzzy soft set $F_\phi \in FS(U, E)$ is said to be null fuzzy soft set and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fuzzy soft set $\overline{0}$ of U, where $\overline{0}(x) = 0$ for all $x \in U$.

2.2 [3].Definition

Let $F_E \in$ FS (U, E) and $F_E(e) = \overline{1}$ for all $e \in E$, where $\overline{1}(x) = 1$ for all $x \in U$. Then F_E is called absolute fuzzy soft set. It is denoted by $\tilde{E}$.

2.3[3].Definition

A fuzzy soft set F_A is said to be a fuzzy soft subset of a fuzzy soft set G_B over a common universe U if $A \subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e \in A$, *i. e.*, if $\mu^e_{F_A}(x) \leq \mu^e_{G_B}(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F_A \tilde{\subseteq} G_B$.

2.4[11].Definition

A fuzzy soft set F_A in a FSTS (U,E,ψ) is called a fuzzy soft nowhere dense set if there exists no non –zero fuzzy soft open set if there exists no non – zero fuzzy soft open set G_B in (U,E,ψ) such that $GB < cl^{fs}(F_A)$. that is, $int^{fs}cl^{fs}(F_A)=0$.

2.5[11].Definition

A FSS F_A is a FSTS (U,E,ψ) is called a fuzzy soft dense if there exists no fuzzy soft closed set G_B in (U,E,ψ), such that $F_A < G_B < 1$, that is $cl^{fs}(F_A) = 1$.

2.6[11].Definition

Let (U,E,ψ) be a fuzzy soft topology. A Fuzzy soft set F_A in (U,E,ψ) is called fuzzy soft first category. If $\bigvee_{i=1}^{\infty}(F_{A_i})$ where $\bigvee_{i=1}^{\infty}(F_{A_i})$, where F_{A_i}'s are fuzzy soft nowhere dense sets in (U,E,ψ). Any other fuzzy soft set in (U,E,ψ) is said to be of fuzzy soft second category.

2.7[12].Definition

Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψsatisfies the following axioms:

- ϕ, $\bar{E}$ belong to ψ.
- The union of any number of fuzzy soft sets in ψ belongs to ψ.
- The intersection of any two fuzzy soft sets ψ belongs to ψ.

The triplet (U, E, φ) is called a fuzzy soft topological space over U. The members of are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

2.8[12].Definition

The union of all fuzzy soft open subsets of $F_A$over (U, E) is called the interior of F_A and is denoted by int^{fs} (F_A).

2.9[12].Proposition

$int^{fs}(F_A \tilde{\wedge} G_B) = int^{fs}(F_A) \tilde{\wedge} int^{fs}(G_B)$.

2.10. [12].Definition

Let $F_A \in$ FS (U, E) be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.

2.11.[12].Remarks

- For any fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ), it is easy to seethat $(cl^{fs}(F_A))^c = int^{fs}(F_A^C)$ and $(int^{fs}(F_A))^c = cl^{fs}(F_A^C)$.
- For any fuzzy softF_A subset of a fuzzy soft topological space (U, E,ψ,), we define the fuzzy soft subspace topology ψ_{F_A} on F_A by $K_D \in \psi_{F_A}$ if $K_D = F_A \tilde{\wedge} G_B$ For some $G_B \in \psi$.
- For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of $H_C$in F_A by $int_{F_A}^{fs}(H_C)$ and $cl_{F_A}^{fs}(H_C)$, respectively.

2.12[2].*Definition*

Let (X ,T ,E) be a fuzzy soft topological space and$f_A \varepsilon$FSOS(X).

- Arbitary fuzzy soft union of fuzzy semi open soft set is fuzzy semi open soft.
- Arbitary fuzzy soft intersection of fuzzy semi closed soft set is fuzzy semi closed soft.

2.14[2].Theorem

Let (X ,T ,E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$

Then:

- $f_A \in FSOS(X)$ if and only if $Fcl(f_A) = Fcl(Fint(f_A))$.
- If $g_B \in$T , then $g_B \tilde{\subset} Fcl(f_A) | Fcl(g_B \tilde{\subset} g_B)$.

2.15[2].Theorem

Let (X ,T ,E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$ .Then:

- $f_A \in FSOS(X)$ if and only if there exists $g_B \in$ T such that $g_B \hat{o} f_A \hat{o} Fcl(g_B)$.
- If $f_A FSOS(X)$ and $f_A \hat{o} h_D \hat{o} Fcl(f_A)$, then $h_D \in FSOS(X)$

2.16[2].Definition

Let (X ,T ,E) be a fuzzy soft topological space, $f_A \in FSS(X)_E$ and$f_e \tilde{\in} FSS(X)_E$. Then

- f_e is calledfuzzy semi interior soft point of f_A if $\exists g_B \in FSOS(X)$ such that $f_e \tilde{\in} g_B \hat{o} f_A$.The set of all fuzzy semi interiorsoft points of f_A is called the fuzzy semi soft interior of f_A and is denoted by $FSint(f_A)$ consequently, $FSint(f_A) = \tilde{\cup}\{g_B : g_B \mid f_A , g_B \in FSOS(X)\}$.

- f_e is called fuzzy semi cluster soft point of $f_A if f_A \widetilde{\cap} h_D \neq \widetilde{0_E} \forall h_D \in FSOS(X)$. The set of all fuzzy semi clusters soft points of $f_A$is called fuzzy semi soft closure of $f_A and$denoted by FS$cl(f_A)$. Consequently, FScl (f_A) = $\widetilde{\cap}$ { $h_D : h_D \in FSCS(X), f_A \setminus h_D$}.

2.17[2]: Theorem

Let (X , T, E) be fuzzy soft topological space and $f_A, g_B \in FSS(X)_E$. Thenthe following properties are satisfied for the fuzzy semi interior operator, denoted by FSint.

- FSint ($\widetilde{1_E}$) = $\widetilde{1_E}$ and FSint ($\widetilde{0_E}$)) = $\widetilde{0_E}$.
- FSint $(f_A) \hat{o} f$ (A.

3. Fuzzy soft regularly nowhere dense sets

3.1.Definition

A Fuzzy soft nowhere dense F_A is called a fuzzy soft regularly nowhere dense set in the FSTS (U,E,ψ), if $F_A \leq cl^{fs}(G_B) \wedge cl^{fs}(H_C)$ and $G_B \leq 1\text{-}H_C$ where $G_B, H_C \in \psi$.

3.1.1.Example

Let U = {a,b,c }. The fuzzy soft sets $F_A, G_B \& H_C$ are defined on U as follows:

F_A: U $\longrightarrow$ [0,1] is defined as $F_A(a) = 0.6, F_B(b) = 0.4, F_B(c) = 0.5$

G_B : U $\longrightarrow$ [0,1] is defined as $G_B(a) = 0.4, G_B(b) = 0.5, G_B(C) = 0.6$

H_C :U$\longrightarrow$ [0,1] is defined as $H_C(a) = 0.6, H_C(b) = 0.6, H_C(c) = 0.6$

Clearly ψ={0, $F_A, G_B, H_C, F_A \vee G_B, F_A \wedge G_B, 1$} is a FST on U .

The fuzzy soft nwd sets in (U,E,ψ) are { 1- G_B ,1- H_C, 1- $F_A \vee G_B$}, but the fuzzy soft regularly nowhere dense sets in (U,E,ψ) are {1- H_C, 1- $F_A \vee G_B$}. The Fuzzy Soft nowhere dense set in (U,E,ψ) are $1-G_B \leq cl(H_C) \wedge cl(G_B)$. Therefore the fuzzy soft regularly nowhere dense sets in (U,E,ψ) are 1- H_C, 1- $F_A \vee G_B$.

3.1.Proposition

If $F_A \leq cl(G_B) \wedge Cl(H_C)$ and $F_A \leq 1\text{-}H_C$ where $G_B, H_C \in \psi$, for a fuzzy soft closed set with $int^{fs}(F_A) = 0$ in a FSTS(U,E,ψ), then F_A is a fuzzy soft regularly nowhere dense set in (U,E,ψ).

Proof:

Suppose that $F_A \leq cl(G_B) \wedge Cl(H_C)$ and $F_A \leq 1\text{-}H_C$ where $F_A, H_C \in T$ .Since $F_A$is a fuzzy soft closed set with $int^{fs}(F_A) = 0$ in (U,E,ψ), then $int^{fs}[cl^{fs}(F_A)] = int^{fs}(F_A)=0$. $\Rightarrow$ F_A is a Fuzzy Soft nowhere dense set in (U,E,ψ) and hence F_A is a fuzzy soft regularly nowhere dense set in (U,E,ψ).

3.2.Proposition

If $F_A \leq cl(G_B) \wedge Cl(H_C)$ and $G_B \leq 1\text{-}H_C$ where $G_B, H_C \in \psi$, for a Fuzzy Soft Closed Set with $int^{fs}(F_A) = 0$ in a FSTS (U,E,ψ), then $cl^{fs}(F_A)$ is a fuzzy soft regularly nwd set in (U,E,ψ).

Proof:

By proposition 3.1, F_A is a fuzzy soft regularly nowhere dense set in (U,E,ψ). since F_A is a FSCS (short form:fuzzy soft closed set)with $int^{fs}(F_A)=0$ in (U,E,ψ),then $cl^{fs}\{int^{fs}[cl^{fs}(F_A)]\}=cl^{fs}[int^{fs}(FA)]=Cl^{fs}(F_A)=F_A=0$[since $cl^{fs}(F_A)=F_A$]. Thus $cl^{fs}(F_A)$ is a Fuzzy soft regularly nowhere dense set in (U,E,ψ).

3.3.Proposition

If $J_A \geq int^{fs}(G_B) \vee int^{fs}(H_C)$ and $G_B \geq 1\text{-}H_C$ where $G_B, H_C \in \psi$, for a fuzzy soft open set with $cl^{fs}(J_A)= 1$ in a FSTS (U,E,ψ), then J_A is a fuzzy soft dense set in (U,E,ψ).

Proof:

Let us assume that $J_A \geq int^{fs}(G_B) \vee int^{fs}(H_C)$ and $G_B \geq 1\text{-}H_C$ where $G_B, H_C \in \psi$, since JA is a fuzzy soft open set with $cl^{fs}(J_A) = 1$ in (U,E,ψ), then $cl^{fs}[int^{fs}(F_A)]= cl^{fs}(F_A)=1$[Since $int^{fs}(J_A) =J_A$]. Thus J_A is a fuzzy soft dense set in (U,E,ψ).

3.4.Proposition

If J_A and K_A are fuzzy soft regularly nowhere dense set in (U,E,ψ), then ($J_A \wedge K_A$) is a fuzzy soft regularly nwd set in (U,E,ψ).

Proof:

Let J_A and K_A are fuzzy soft closed sets with $int^{fs}(J_A)=0$ and $int^{fs}(K_A)=0$, then $int^{fs}[cl^{fs}(J_A)]=int^{fs}(J_A)=0$, and $int^{fs}[cl^{fs}(K_A)]= int^{fs}(K_A)=0$ [Since $cl^{fs}(J_A) =J_A$; $Cl^{fs}(K_A)= K_A$]. Now $J_A \wedge K_A$

$\leq int^{fs}$ [$cl^{fs}(J_A)$] $\wedge$ $Cl^{fs}(K_A)$] $\leq int^{fs}$[$cl^{fs}(J_A)$]$\wedge$[int^{fs}[$cl^{fs}(K_A)$]=0$\wedge$0=0. Thus $J_A \wedge K_A$ is a fuzzy soft nowhere dense set in (U,E,ψ), which implies that ($J_A \wedge K_A$) is a fuzzy soft regularly nowhere dense set in (U,E,ψ).

3.5.Definition

Let (U,E,ψ) be a FSTS. A fuzzy soft set F_A in (U,E,ψ) is called a fuzzy soft regularly first category set, if $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$, where ($F_{A_i}$)'s are FSRnwds in (U,E,$\psi$). Any other FSS in (U ,E,ψ) is said to be of [short form fuzzy soft regularly FSR]FSR second category.

3.5.Proposition

If F_A is a FSRFCS in the FSTS (U,E,ψ), then $1-F_A = \bigwedge_{i=1}^{\infty}(G_{B_i})$,where $cl^{fs}[int^{fs}(G_{B_i})]=1$ in (U,E,ψ).

Proof:

Let F_A be a FSRFCS in (U,E,ψ), then $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$, where ($\bigvee_{i=1}^{\infty}(F_{A_i})$)'s are FSRnwd set in (U,E,ψ).since (F_{A_i})'s are fuzzy soft regularly nowhere dense sets in (U,E,ψ), then $1-(F_{A_i})$' s are fuzzy soft open sets with $cl^{fs}(G_{B_i})=1$ in (U,E,ψ). Let $1-(F_{A_i}) = (G_{B_i})$, and hence $1-F_A = 1-\bigvee_{i=1}^{\infty}(F_{A_i}) = \bigwedge_{i=1}^{\infty}(1-F_{A_i}) = \bigwedge_{i=1}^{\infty}(G_{B_i})$, where $cl^{fs}\{int^{fs}[\bigwedge_{i=1}^{\infty}(G_{B_i})]\} = cl^{fs}[int^{fs}(G_{B_i})] = cl^{fs}(G_{B_i}) = 1$.

3.6.Definition

Let F_A be a FSRFCS in a FSTS (U,E,ψ), then $1-F_A$ is called a fuzzy soft residual set in (U,E,ψ).

3.6.Proposition

If F_A is a FSRFCS in the FSTS (U,E,ψ), then $1-F_A$ is a FSR residual set in (U,E,ψ).

Proof:

Let F_A be a FSRFCS in (U,E,ψ), then $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$, where ($F_{A_i}$)'s are FSR nowhere dense set in (U,E,ψ). By proposition 3.5, $1-F_A$ is a fuzzy soft regularly residual set in (U,E,ψ).

3.7.Proposition

If F_A is a fuzzy soft residual set in (U,E,ψ), then F_A is a FSR residual set in (U,E,ψ).

Proof:

Let F_A be a fuzzy soft residual set in (U,E,ψ), then $F_A = \bigwedge_{i=1}^{\infty}(F_{A_i})$ where $cl^{fs}[int^{fs}(F_{A_i})]=1$.since F_{A_i} is a fuzzy soft open set in (U,E,ψ), then int^{fs} (F_{A_i}) = F_{A_i} .Now ,$cl^{fs}[int^{fs}(F_{A_i})]= cl^{fs}[\bigwedge_{i=1}^{\infty} int^{fs}(F_{A_i})] = cl^{fs}\{ int^{fs}[\bigwedge_{i=1}^{\infty}(F_{A_i})]\} = cl^{fs}int^{fs}(F_{A_i}) = cl^{fs}(F_{A_i}) = 1$. Thus F_A is a FSR residual set in (U,E,ψ).

3.8.Proposition

If F_A is a fuzzy soft semi closed set with $int^{fs}(F_A) \neq 0$ in a FSTS (U,E,ψ), then $int^{fs}[cl^{fs}(F_A)] \neq 0$.

Proof:

Let F_A be a fuzzy soft semi – closed set with $int^{fs}(F_A) \neq 0$ in (U,E,ψ), then $int^{fs}[cl^{fs}(F_A)] \leq F_A$ implies that $int^{fs}[cl^{fs}(F_A)] \leq int^{fs}(F_A) \neq 0$.

3.9.Proposition

If F_A is a fuzzy soft semi – closed set with $int^{fs}(F_A) \neq 0$ in a FSTS (U,E,ψ), then F_A is not a fuzzy soft nowhere dense set in (U,E,ψ).

Proof:

Let F_A be a fuzzy soft semi- closed set with $int^{fs}(F_A) \neq 0$ in (U,E,ψ). By proposition 3.8, $int^{fs}[cl^{fs}(F_A)] \neq 0$, implies that F_A is not a fuzzy soft nowhere dense set in (U,E,ψ).

3.10.Proposition

If F_A is a fuzzy soft closed set with $int^{fs}(F_A) \neq 0$ in a FSTS (U,E,ψ), then $int^{fs}(F_A) \neq 0$ in a FSTS (U,E,ψ),then $int^{fs}[cl^{fs}(F_A)] \neq 0$

Proof:

Let F_A be a fuzzy soft closed set with $int^{fs}(F_A) \neq 0$ in (U,E,ψ). Since $cl^{fs}(F_A) = F_A$, then $int^{fs}[cl^{fs}(F_A)] \leq int^{fs}(F_A) \neq 0$.

3.11.,Proposition

If F_A is a fuzzy soft closed set with $int^{fs}(F_A) \neq 0$ in a FSTS (U,E, ψ), then F_A is not a fuzzy soft nowhere dense set in (U,E, ψ).

Proof:

Let F_A be a fuzzy soft closed set with $int^{fs}(F_A) \neq 0$ in (U,E, ψ). By proposition 3.10, $int^{fs}(F_A) \neq 0$, implies that then F_A is not a fuzzy soft nowhere dense set in (U,E, ψ).

3.12.Proposition

If $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$, where ($F_{A_i}$)'s are fuzzy soft closed set with $int^{fs}(F_A) \neq 0$ in a FSTS (U,E, ψ), then F_A is not a fuzzy soft nowhere dense set in (U,E, ψ).

Proof:

Let $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft closed set with $int^{fs}(F_A) \neq 0$. Since F_A is a fuzzy soft closed set in (U,E, ψ), then by proposition 3.11, F_A is not a fuzzy soft nowhere dense set in (U,E, ψ).

3.13.Definition

Let (U,E, ψ) be a FSTS . A FSS [fuzzy soft set short form] F_A in (U,E, ψ) is called a FSR residual set, if $F_A = \Lambda_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are such that $cl^{fs}[int^{fs}(F_{A_i})]=1$ in (U,E,ψ).

3.14.Definition

A fuzzy soft dense G_B is called a FSR dense set in the FSTS (U,E,ψ), if $G_B \geq int(H_C) \vee int(J_D)$ and $H_C \geq 1 - K_E$ where $H_C, J_D \in \psi$.

3.15,Proposition

If $F_A$is a fuzzy soft closed set with $int^{fs}(F_A)= 0$ in a FSTS (U,E,ψ), then F_A is a FSR nowhere dense set in (U,E,ψ).

Proof:

Let F_A is a fuzzy soft closed set with $int^{fs}(F_A) \neq 0$ in (U,E,ψ). Since F_A is a fuzzy soft closed in (U,E,ψ) , then $cl^{fs}(F_A) = F_A$ implies that $int^{fs}[cl^{fs}(F_A)] = int^{fs}(F_A)=0$. Thus F_A is a fuzzy soft nowhere dense set and hence F_A is a fuzzy soft residual nowhere dense set in (U,E,ψ).

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On Fuzzy Soft Strongly Baire Spaces

E. Poongothai, S. Divyapriya

Abstract

In this present paper ,we have introduced and studied the concept of FSSNWDS , FSSFCS, FSSSCS, FSSRS, in FSTS . The notion of FSSBS is defined and sundry characterizations and properties are probe in this work . Finally we give an some related examples in this paper.

Keywords:

Fuzzy soft dense sets[FSDS], Fuzzy soft strongly nowhere dense sets[FSSNWDS], Fuzzy soft simply open sets[FSSOPS],FSSBS.

1.Introduction

The concept of fuzzy topological space was introduced by **C.L. Chang** [4] in 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Several Mathematicians have tried all the pivotal concepts of general topology for extension to the fuzzy setting. In 1965, **Lotfi A.Zadeh** introduced the concept of fuzzy sets as a new approach for modeling uncertainties[19].In 1899, **Rene Louis Baire** [2] introduced the concepts of first and second category sets in his doctoral thesis. In classical topology, Baire space, named in honor of Rene Louis Baire, was first introduced in **Bourbaki's** [**3**] Topologie generale Chapter IX. The concepts of Baire spaces have been studied extensively in classical topology in [**5,12,20,21**]. The concept of Baire spaces in fuzzy setting was introduced and studied by **G.Thangaraj and S.Anjalmose** in [15].**Maji,P.K.Roy**,[10] further studied the theory of soft sets and used this theory to solve some decision making problems. The concept of fuzzy soft set is introduced and studied [6-9,12] a more generalized concept which is a combination of fuzzy set and fuzzy soft set. The notion of fuzzy simply open sets by means of fuzzy nowhere denseness of fuzzy boundary sets in fuzzy topological spaces is introduced and studied by **G.Thangaraj and K.Dinakaran** in [**16**]. In this paper the concept of fuzzy soft strongly Baire spaces in FSTS are introduced and studied.

2.Preliminaries

In section 2 we have given some basic definitions and notion are self-contained.

Definition 2.1[6]:

The fuzzy soft set $F_\phi \in FS(U, E)$ is said to be null fuzzy soft set and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fuzzy soft set $\bar{0}$ of U, where $\bar{0}(x) = 0$ for all $x \in U$.

Definition 2.2 [6]:

Let $FE \in FS(U, E)$ and $FE(e) = \bar{1}$ all $e \in E$, where $\bar{1}(x) = 1$ for all $x \in U$.Then F_E is called absolute fuzzy soft set. It is denoted by $\bar{E}$.

Definition 2.3 [6]:

A fuzzy soft set F_A is said to be a fuzzy soft subset of a fuzzy soft set G_B over a common universe U if $A \subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e \in A$, *i.e.*, if $\mu^e F_A(x) \leq \mu^e G_B(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F_A \tilde{\subseteq} G_B$.

Definition 2.4[6]:

Two fuzzy soft sets F_A and G_B over a common universe U are said to be fuzzy soft equal if F_A is a fuzzy soft subset of G_B of G_B is a fuzzy soft subset of F_A.

Definition 2.5[10]:

The union of two fuzzy soft sets F_A and G_B over the common universe U is the fuzzy soft set HC, defined by$H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$for all$e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \bar{\vee} G_B$

Definition 2.6[6]:

Let F_A and G_B be two fuzzy soft set, then the intersection of F_A and G_B is a fuzzy soft set H_C, defined by $H(e) = \mu^e_{F_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \bar{\wedge} G_B$.

Lemma 2.1[1]:

For a family A = $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\vee(\text{cl}(\lambda_\alpha)) \leq \text{cl}(\vee(\{\lambda_\alpha)$. In case A is a finite set, $\vee(\text{cl}(\lambda_\alpha)) = \text{cl}(\vee((\lambda_\alpha))$. Also $\vee(\text{int}(\lambda_\alpha)) \leq \text{int}(\vee(\lambda_\alpha))$.

Definition 2.7[18]:

Let $F_A \in \text{FS}(U, \text{E})$ be a fuzzy soft set. Then the complement of F_A, denoted byF^C_A,defined by

$$F^C_A(e) = \begin{cases} \bar{1} - \mu^e_{F_A}, & if \ e \in A \\ \bar{1}, & if e \notin A \end{cases}$$

Definition 2.8[14]:

Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:

(i)$\phi, \bar{E}$belong to ψ.

(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ.

(iii) The intersection of any two fuzzy soft sets ψ belongs to ψ. The triplet (U, E, ψ) is called a fuzzy soft topological space over U. The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

Definition 2.9[14]:

The union of all fuzzy soft open subsets of F_A over (U, E, ψ) is called the interior of F_A and is denoted by$int^{fs}(F_A)$.

Proposition 2.1[14]:

$int^{\text{fs}}(F_A \bar{\wedge} G_B) = int^{f\text{S}}(F_A) \bar{\wedge} int^{fs}(G_B)$.

Definition 2.10 [14]:

Let$F_A \in \text{FS}(U, E)$ be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.

Remarks 2.11 [14]:

(1)For any fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ), it is easy to see that $((F_A))^c = int^{fs}(F^C_A)$ and $(int^{fs}(F_A))^c = cl^{fs}(F^C_A)$.

(2) For any fuzzy soft F_A subset of a fuzzy soft topological space (U, E, ψ) we define the fuzzy soft subspace topology on F_A by $K_D \in \psi_{F_A}$if$K_D = F_A \bar{\wedge} G_B$ for some $G_B \in \psi$.

(3)For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of H_C in F_A by $int^{fs}_{F_A}(H_C)$ and $cl^{fs}_{F_A}(H_C)$, respectively.

Definition2.12[13]:

A fuzzy soft set F_A in a FSTS $(U, E, \psi)$is called a FSNWD set if there exist no non-zero fuzzy soft open set G_B in (U,E, ψ) such that $G_B < \text{cl}^{fs}(F_A)$. ie), $\text{int}^{fs}\text{cl}^{fs}(F_A) = 0$.

Definition 2.13[13]

A fuzzy soft set F_A in a FSTS $(U, E, \psi)$is called fuzzy soft dense if there exist no fuzzy soft closed set G_B in (U, E, ψ) such that $F_A < G_B < 1$. ie) $\text{cl}^{fs}(F_A) = 1$.

Definition 2.14[13]:

Let (U, E, ψ) be a fuzzy soft topology . A fuzzy soft set F_A in $(U, E, \psi)$is called fuzzy soft first category . If $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$ where (F_{A_i})'s are fuzzy soft nowhere dense sets in (U, E, ψ). Any other fuzzy soft set in (U, E, ψ) is said to be of fuzzy soft second category.

3. FUZZY SOFT STRONGLY NOWHERE DENSE SETS

Definition3.1:

A FSS F_A defined on U is called a FSSNWDS in FSTS (U, E,ψ).If $F_A \wedge (1-F_A)$ is a FSNWDS in (U,E,ψ). ie) F_A is a FSSNWDS in (U,E,ψ).If $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$ in (U,E,ψ).

Example3.1:

Let U = {a,b,c }. The fuzzy soft sets F_A, G_B & H_C are defined on U as follows:

F_A: U $\longrightarrow$ [0,1] is defined as F_A (a) =0.4, F_B(b) = 0.5, F_B(c) = 0.3

G_B : U $\longrightarrow$ [0,1] is defined as G_B (a) = 0.5, G_B (b) = 0.5, G_B (C) =0.3

H_C :U $\longrightarrow$ [0,1] is defined as H_C(a) =0.4, H_C(b) = 0.4, H_C(c) = 0.4

Clearly ψ ={0, F_A, G_B , H_C, $G_B \vee H_C$, $H_C \vee F_A$, $G_B \wedge H_C$, 1} is a FST on U

Hence $int^{fs}cl^{fs}(F_A) = int^{fs}(1-G_B \vee H_C)=0$ in (U,E,ψ) and hence (U,E,ψ) is a FSSNWDS .

Proposition 3.1:

If F_A is a FSNWDS in a FSTS (U,E,ψ), after that F_A is a FSSNWDS in (U,E,ψ).

Proof:

Let F_A be a FSNWDS in (U,E,ψ). Subsequently $int^{fs}cl^{fs}(F_A) = 0$, in (U,E,ψ). since $[F_A \wedge (1-F_A)] \leq F_A$, in (U,E,$\psi$), $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq int^{fs}cl^{fs}(F_A)$ and hence $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq 0$ .ie), $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$. Hence F_A is a FSSNWDS in (U,E,ψ).

Remark 3.1:

A FSSNWDS in a FSTS (U,E,ψ) be a not required FSNWDS in (U,E,ψ). For in example 3.1, H_C is a FSSNWDS , but not a FSNWDS in (U,E,ψ).

Proposition 3.2:

If $int^{fs}cl^{fs}(F_A)$ is a fuzzy soft dense set, for a fuzzy soft set F_A defined on U, in a FSTS (U,E,ψ), then F_A is a FSSNWDS in (U,E,ψ).

Proof:

Suppose that $int^{fs}(F_A)$ is a fuzzy soft dense set in (U,E,ψ). Then $cl^{fs}[int^{fs}(F_A)] = 1$ in (U,E,ψ) and $1 - cl^{fs}[int^{fs}(F_A)] = 0$. $\Rightarrow int^{fs}cl^{fs}(1- F_A) = 0$ in (U,E,ψ). since $F_A \wedge (1-F_A) \leq (1-F_A)$, $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq int^{fs}cl^{fs}(1- F_A)$ and hence $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq 0$. i.e)$int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$. Hence F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.3:

If $1-F_A$ is a FSNWDS in a FSTS (U,E,ψ), then F_A is a FSSNWDS in (U,E,ψ).

Proof:

Suppose that $1-F_A$ is a FSNWDS in (U,E,ψ). Then, $int^{fs}[cl^{fs}(1-F_A)] = 0$ in (U,E,ψ).Since $F_A \wedge (1-F_A) \leq 1-F_A$, $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq int^{fs}[cl^{fs}(1- F_A)]$ and hence $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq 0$. ie) $int^{fs}cl^{fs}[F_A \wedge (1-F_A)]=0$ in (U,E,ψ) and hence F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.4:

If $cl^{fs}[int^{fs}(1- F_A)]=1$, for a fuzzy soft set F_A defined on U in a FSTS (U,E,ψ), then F_A is a FSSNWDS in (U,E,ψ).

Proof:

Suppose that $cl^{fs}[int^{fs}(1-F_A)]=1$ in (U,E,ψ). Then $1- cl^{fs}[int^{fs}(1-F_A)] = 0$ and $1-\{1-int^{fs}[cl^{fs}(F_A)]\} = 0 \Rightarrow int^{fs}[cl^{fs}(F_A)]=0$ in (U,E,ψ). Thus F_A is a FSNWDS in (U,E,ψ). Then by proposition 3.1, F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.5:

If F_A is a FSSNWDS in a FSTS(U,E,ψ), then $1- F_A$ is also a FSSNWDS in (U,E,ψ).

Proof:

Let $F_A$be a FSSNWDS in (U,E,ψ). Then $int^{fs}cl^{fs}[F_A \wedge (1-F_A)]=0$ in (U,E,ψ). Now $int^{fs}cl^{fs}\{(1-F_A) \wedge [1-(1-F_A)]\} = int^{fs}cl^{fs}([(1-F_A) \wedge F_A]$ and hence $int^{fs}cl^{fs}\{(1- F_A) \wedge [1- (1-F_A)]\} = 0$. $\Rightarrow$ $1- F_A$ is a FSSNWDS in (U,E,ψ).

Proposition 3.6:

If F_A is a FSSNWDS in a FSTS(U,E,ψ), then 1- F_A is a FSSNWDS in (U,E,ψ).

Proof:

Let $F_A$be a FSNWDS in (U,E,ψ). Then, by proposition 3.1, F_A is a FSSNWDS in (U,E,ψ).and by proposition 3.5, 1- F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.7:

If F_A is a FSSNWDS in a FSTS (U,E,ψ), then $cl^{fs}(F_A) \vee cl^{fs}(1-F_A) = 1$, in (U,E,$\psi$).

Proof:

Let $F_A$be a FSSNWDS in (U,E,ψ). Then $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$ in (U,E,ψ). Now $1-int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 1$ and hence $cl^{fs}int^{fs}[\ 1-\{F_A \wedge (1-F_A)\}] = 1$, in (U,E,ψ). But , $cl^{fs}int^{fs}[1-\{F_A \wedge (1-F_A)\}] \leq cl^{fs}[\ 1-\{F_A \wedge (1-F_A)\}]$ implies that $1 \leq cl^{fs}[\ 1-\{F_A \wedge (1-F_A)\}]$. Thus , $cl^{fs}[\ 1-\{F_A \wedge (1-F_A)\}] = 1$, this implies that $cl^{fs}[(1-F_A) \vee F_A] = 1$ in (U,E,ψ). But by lemma 2.1, $cl^{fs}[(1-F_A) \vee F_A] = cl^{fs}(1-F_A) \vee cl^{fs}(F_A)$. Hence $cl^{fs}(F_A) \vee cl^{fs}(1-F_A) = 1$, in (U,E,ψ).

Proposition3.8:

If F_A is a FSS[fuzzy soft set FSS] defined on U such that $int^{fs}[fsbd(F_A)] = 0$ in a FSTS (U,E,ψ), then F_A is a FSSNWDS in(U,E,ψ).

Proof:

Let $F_A$be a FSS defined on U .such that $int^{fs}[fsbd(F_A)] = 0$ in (U,E,ψ). since $fsbd(F_A) = cl^{fs}(F_A) \wedge cl^{fs}(1-F_A)$ and $cl^{fs}(F_A) \wedge cl^{fs}(1-F_A) \geq cl^{fs}[F_A \wedge (1-F_A)]$, we have $fsbd(F_A) \geq cl^{fs}[F_A \wedge (1-F_A)]$ and hence $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq int^{fs}[fsbd(F_A)]$ in (U,E,ψ). then $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq 0$. ie) $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$ and hence F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.9:

If F_A is a FS simply open set in a FSTS (U,E,ψ), then F_A is a FSSNWDS in(U,E,ψ).

Proof:

Let F_A be a FS simply open set in (U,E,ψ). Then $int^{fs}cl^{fs}[fsbd(F_A)] = 0$, in (U,E,$\psi$). But $int^{fs}[fsbd(F_A)] \leq int^{fs}cl^{fs}[fsbd(F_A)] \Rightarrow int^{fs}[fsbd(F_A)] = 0$ in (U,E,ψ). Then, by proposition 3.8, F_A is a FSSNWDS in(U,E,ψ).

Proposition 3.10:

If F_A is a FSCS with $int^{fs}(F_A) = 0$ in a FSTS (U,E,ψ), then F_A is aFSSNWDS in(U,E,ψ).

Proof:

Let $F_A$be a FS closed set with $int^{fs}(F_A) = 0$ in (U,E,ψ).Then,$int^{fs}cl^{fs}[cl^{fs}(F_A) \wedge cl^{fs}(1-F_A)\ ] = int^{fs}cl^{fs}[F_A \wedge (1- int^{fs}\ F_A)] = int^{fs}cl^{fs}[F_A \wedge 1] = int^{fs}cl^{fs}(F_A) = int^{fs}(F_A) = 0$, and hence F_A is a fuzzy soft simply open set in (U,E,ψ).By prop 3.9,F_A is a FSNWDS in (U,E,ψ).

Proposition 3.11:

If F_A is a FS open and fuzzy soft dense set in a FSTS (U,E,ψ), then F_A is a is a FSSNWDS in (U,E,ψ).

Proof:

Let $F_A$be a fuzzy soft open and fuzzy soft dense set in (U,E,ψ). Then 1-$F_A$is a fuzzy soft closed set with $int^{fs}(1-F_A) = 1-cl^{fs}(F_A) = 1-1 = 0$ in (U,E,ψ). Then ,by proposition 3.10, 1-$F_A$is a FSSNWDS in (U,E,ψ) and by proposition 3.5,1-(1-F_A) is a FSSNWDS in (U,E,ψ) and thus F_A is a FSSNWDS in (U,E,ψ).

Theorem 3.1[17]:

If F_A is a FSNWDS in a FSTS (U,E,ψ) after that $cl^{fs}(F_A)$ is a FSSNWDS in (U,E,ψ).

Proposition 3.12:

If F_A is a FSNWDS in a FSTS (U,E,ψ), then $cl^{fs}(F_A)$ is a FSSOPS in (U,E,ψ).

Proof:

Let $F_A$be a FSNWDS in (U,E,ψ). Then by theorem 3.1, $cl^{fs}(F_A)$ is a fuzzy soft simply open set in (U,E,ψ) and then proposition 3.9, $cl^{fs}(F_A)$ is a FSSNWDS in (U,E,ψ).

Definition 3.2:

A FSS $F_A$in a FSTS (U,E,ψ) is called a FSS first category set if $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). Any other FSS in (U,E,ψ) is said to be a FSS second category set in (U,E,ψ).

Definition 3.3:

If F_A is a FSS first category set in a FSTS (U,E,ψ), then 1-F_A is a fuzzy soft strongly residual set in (U,E,ψ).

Definition 3.4:

A FSTS (U,E,ψ) is called a FSS first category set, if the FSS 1_x is a FSS first category set in (U,E,ψ). ie), $1_x = \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). otherwise (U,E,ψ) will be called a FSS second category set.

Proposition 3.13

If F_A is a fuzzy soft first category set in a FSTS (U,E,ψ). Then F_A is a FSS first category set in (U,E,ψ).

Proof:

Let F_A be a fuzzy soft first category set in (U,E,ψ). Then $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSNWDS in (U,E,ψ). By proposition 3.1, the FSNWDS. Where (F_{A_i})'s are FSSNWDS in (U,E,ψ) and hence $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). $\Rightarrow F_A$ is a FSS first category set in (U,E,ψ).

Proposition 3.14

If $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft closed set with $int^{fs}(F_{A_i}) = 0$ in (U,E,ψ), then $F_A$is a FSS first category set in (U,E,ψ).

Proof:

Suppose that $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where $1\text{-}F_{A_i} \in T$ and $int^{fs}(F_{A_i}) = 0$ in (U,E,ψ). Now by proposition 3.10, the fuzzy soft closed sets (F_{A_i})'s with $int^{fs}(F_{A_i}) = 0$, are FSSNWDS in (U,E,ψ) and hence $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ),$\Rightarrow F_A$ is a FSS first category set in (U,E,ψ).

Proposition 3.15

If F_A is a fuzzy soft $F_{\sigma-}$ set such that $int^{fs}(F_A) = 0$ in (U,E,ψ), then F_A is a FSSFCS in (U,E,ψ).

Proof:

Let F_A be a fuzzy soft $F_{\sigma-}$ set in (U,E,ψ). Then $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSCS in (U,E,ψ). By lemma 2.2, $\bigvee_{i=1}^{\infty} i int^{fs}(F_{A_i}) \leq int^{fs}\{\bigvee_{i=1}^{\infty}(F_{A_i})\}$, in (U,E,$\psi$). $\Rightarrow \bigvee_{i=1}^{\infty} i int^{fs}(F_{A_i}) \leq int^{fs}\{\bigvee_{i=1}^{\infty}(F_{A_i})\} = int^{fs}(F_A)$. since $int^{fs}(F_A)= 0$, $\bigvee_{i=1}^{\infty} i int^{fs}(F_{A_i}) = 0$ in (U,E,ψ). $\Rightarrow int^{fs}(F_{A_i}) = 0$, thus (F_{A_i})'s are FSCS with $int^{fs}(F_{A_i}) = 0$, in (U,E,ψ). Then by prop 3.10, (F_{A_i})'s areFSSNWDS in (U,E,ψ). Hence $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). $\Rightarrow F_A$ is a FSSFCS in (U,E,ψ).

Proposition 3.16:

If F_A is a FSσ $-$NWDS in a FSTS (U,E,ψ) then F_A is a FSSFCS in (U,E,ψ).

Proof:

Let F_A be a FS σ $-$NWDS in (U,E,ψ). Then F_A is a fuzzy soft $F_{\sigma-}$ set in (U,E,ψ). such that $int^{fs}(F_A) = 0$. Then by proposition 3.15, F_A is a FSSFCS in (U,E,ψ).

Proposition 3.17:

If $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft simply open sets in a FSTS (U,E,ψ), then F_A is a FSSFCS in (U,E,ψ).

Proof:

Suppose that $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft simply open sets in (U,E,ψ). By proposition 3.9, the fuzzy soft simply open set (F_{A_i})'s are FSSNWDS in (U,E,ψ) and hence F_A is a FSSFCS in (U,E,ψ).

Proposition 3.18:

If $F_A= \bigvee_{i=1}^{\infty}(F_{A_i})$, where $int^{fs}[fsbd(F_{A_i})] = 0$ in (U,E,ψ), then F_A is a FSSFCS in (U,E,ψ).

Proof:

The proof follows from proposition 3.8.

Proposition 3.19:

If $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft open and fuzzy soft dense set in a FSTS (U,E,ψ), thenF_A is a FSSFCS in (U,E,ψ).

Proof:

The proof follows from proposition 3.11.

Proposition 3.20:

If F_A is FSFCS in a FSTS (U,E,ψ), then there exist a FSSFCS G_B in (U,E,ψ) such that $cl^{fs}(F_A) \geq G_B$.

Proof:

Let F_A be a FSFCS in (U,E,ψ). Then $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft nowhere dense set in (U,E,ψ). By proposition 3.12,{ $cl^{fs}(F_{A_i})$}'s are FSSNWDS in (U,E,ψ). Then $\bigvee_{i=1}^{\infty} cl^{fs}(F_{A_i})$ is a FSSFCS in (U,E,ψ). Let $G_B = \bigvee_{i=1}^{\infty} cl^{fs}(F_{A_i})$. Now $\bigvee_{i=1}^{\infty} cl^{fs}(F_{A_i}) \leq cl^{fs}[\bigvee_{i=1}^{\infty}(F_{A_i})] \Rightarrow G_B \leq cl^{fs}(F_A)$, in (U,E,ψ).

4.FUZZY SOFT STRONGLY BAIRE SPACE

Definition 4.1:

A FSTS (U,E,ψ) is called a FSSBS if $cl^{fs}[\bigvee_{i=1}^{\infty}(F_{A_i})] = 1$ where (F_{A_i})'s are FSSNWDS in (U,E,ψ).

Example 4.1.1:

Let U = {a,b,c}. The fuzzy soft sets F_A,G_B &H_C are defined on U as follows:

F_A: U $\longrightarrow$ [0,1] is defined as $F_A(a) = 0.2$, $F_B(b) = 0.5$, $F_B(c) = 0.4$

G_B : U $\longrightarrow$ [0,1] is defined as $G_B(a) = 0.4$, $G_B(b) = 0.5$, $G_B(C) = 0.2$

H_C :U $\longrightarrow$ [0,1] is defined as $H_C(a) = 0.5$, $H_C(b) = 0.3$, $H_C(c) = 0.3$

Then T ={0, F_A, G_B, H_C, $F_A \vee G_B$, $G_B \vee H_C$, $H_C \vee F_A$, $F_A \wedge G_B$, $G_B \wedge H_C$, $H_C \wedge F_A$, 1} is a fuzzy topology on U.

The fuzzy strongly nowhere dense set are in (U,E,ψ) are{1- F_A,1- G_B , 1-H_C,1-$F_A \vee G_B$,1- $G_B \vee H_C$,1- $H_C \vee F_A$, 1-$F_A \wedge G_B$, 1-$G_B \wedge H_C$,1-$H_C \wedge F_A$}. $int^{fs}cl^{fs}(1\text{-}H_C \wedge F_A) = intcl^{fs}(\bigvee_{i=1}^{\infty}(1\text{-}H_C \wedge F_A) \vee (H_C \wedge F_A) = int^{fs}cl^{fs}(1\text{-}H_C \wedge F_A) = 0$ .Hence (U,E,ψ) is a fuzzy soft strongly Baire space.

Proposition 4.1:

Let (U,E,ψ) be a FSTS .Then, the following are equivalent.

(i)(U,E,ψ) is a FSSBS.

(ii)$cl^{fs}(F_A) = 1$, for each FSSFCS F_A in (U,E,ψ).

(iii) $int^{fs}(G_B) = 0$, for each fuzzy soft strongly residual set G_B in (U,E,ψ).

Proof:

(i)$\Rightarrow$(ii)

Let F_A be a FSSFCS in (U,E,ψ). Then $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft strongly nowhere dense set in (U,E,ψ). since (U,E,ψ) is a FSSBS , $cl^{fs}[\bigvee_{i=1}^{\infty}(F_{A_i})] = 1$, and hence $cl^{fs}(F_A) = 1$, in (U,E,ψ).

(ii)$\Rightarrow$(iii)

Let G_B be a FSSRS in (U,E,ψ). Then 1- G_B is a FSSFCS in (U,E,ψ). By hypothesis , $cl^{fs}(1\text{-}G_B) = 1$ in (U,E,ψ). Then 1- $int^{fs}(G_B) = cl^{fs}(1\text{-}G_B) = 1$ and hence $int(G_B) = 0$ in (U,E,ψ).

(iii)$\Rightarrow$(i)

Let F_A be a FSSFCS in (U,E,ψ). Then $F_A = \bigvee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). Since F_A is a FSSFCS in (U,E,ψ), 1-F_A is a FSS residual set in (U,E,ψ). By hypothesis , $int^{fs}(1\text{-}F_A) = 0$ in (U,E,ψ). Now 1-$cl^{fs}(F_A) = int^{fs}(1\text{-}F_A) = 0 \Rightarrow cl^{fs}(F_A) = 1$ and then $cl^{fs}[\bigvee_{i=1}^{\infty}(F_{A_i})] = 1$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ) $\Rightarrow$(U,E,ψ) is a FSSBS.

Proposition 4.2:

If (U,E,ψ) is a FSSBS, then

(i)$cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where 1- $F_{A_i} \in T$ and $int^{fs}(F_{A_i}) = 0$ in (U,E,ψ).

(ii)$cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$,where (F_{A_i})'s are fuzzy soft simply open sets in (U,E,ψ).

(iii)$cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i}] = 1$, where $int^{fs}[fsbd(F_{A_i})] = 0$ in (U,E,ψ).

Proof:

(i)Let $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft closed sets with $int^{fs}(F_{A_i}) = 0$. Then , by proposition 3.14, F_A is a FSSFCS in (U,E,ψ). Since (U,E,ψ) is a FSSBS , by proposition 4.1, $cl^{fs}(F_A) = 1$ in (U,E,ψ). Thus $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where 1- $F_{A_i} \in T$ and $int^{fs}(F_{A_i}) = 0$.

(ii) Let $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft simply open sets in (U,E,ψ). Then , by prop 3.17, F_A is a FSSFCS in (U,E,ψ). since (U,E,ψ) is a FSSBS, by proposition 4.1, $cl^{fs}(F_A) = 1$ in (U,E,ψ). Thus $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where (F_{A_i})'s are fuzzy soft simply open sets in a FSSBS (U,E,ψ).

(iii)Let $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft sets on (U) with $int^{fs}[fsbd(F_{A_i})] = 0$ in (U,E,ψ). Then by prop 3.18, F_A is a FSSFCS in (U,E,ψ). Since (U,E,ψ) is a FSSBS, by prop 4.1, $cl^{fs}(F_A) = 1$ in (U,E,ψ). Thus $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where $int^{fs}[fsbd(F_{A_i})] = 0$ in (U,E,ψ).

Proposition 4.3:

If each fuzzy soft open sets is a FSDS in a FSTS (U,E,ψ),then (U,E,ψ) is a FSSBS.

Proof:

Let (F_{A_i})'s are fuzzy soft open sets in (U,E,ψ). By hypothesis (F_{A_i})'s are fuzzy soft dense sets in (U,E,ψ). Then (F_{A_i})'s are fuzzy soft open and fuzzy soft dense sets in (U,E,ψ). Then by prop 3.11, (F_{A_i})'s are FSSNWDS in (U,E,ψ). Let $F_A = \vee_{i=1}^{\infty}(F_{A_i})$. Then F_A is a FSSFCS in (U,E,ψ). Now $cl^{fs}(F_A) = cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] \geq \vee_{i=1}^{\infty} cl^{fs}(F_{A_i}) = \vee_{i=1}^{\infty}(1) = 1$ ie) $cl^{fs}(F_A) = 1$ in (U,E,ψ). Then, by prop 4.1, $(U,E,\psi)$is a FSSBS.

Proposition 4.4:

If F_A is a FSFCS in a FSSBS, then F_A is a FSDS in (U,E,ψ).

Proof:

Let F_A be a FSFCS in (U,E,ψ). By prop 3.17, there exist a FSSFCS $G_B$in (U,E,ψ). such that $cl^{fs}(F_A) \geq G_B$. Then $cl^{fs}[cl^{fs}(F_A)] \geq cl^{fs}(G_B)$. Since (U,E,ψ) is a FSSBS by prop 4.1, for the FSSFCS G_B in (U,E,ψ), $cl^{fs}(G_B) = 1$. Then $cl^{fs}(F_A) \geq 1$. Hence F_A is a FSDS in (U,E,ψ).

Proposition 4.5:

If (F_{A_i})'s (i=1to∞) are fuzzy soft simply open sets in a FSSBS, then (F_{A_i})'s are not FSDS in (U,E,ψ).

Proof:

Let (F_{A_i})'s (i=1to∞) be fuzzy soft simply open sets in (U,E,ψ).Suppose that $F_A = \vee_{i=1}^{\infty}(F_{A_i})$ .Then by prop 3.17, F_A is a FSSFCS in (U,E,ψ).Since (U,E,ψ) is a FSSBS, by prop 4.1, $cl^{fs}(F_A) = 1$ in (U,E,ψ). Then $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$ in (U,E,ψ). But $\vee_{i=1}^{\infty} cl^{fs}(F_{A_i}) < cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] \Rightarrow \vee_{i=1}^{\infty} cl^{fs}(F_{A_i}) \neq 1$ in (U,E,ψ) and hence $cl^{fs}(F_{A_i}) \neq 1$. Thus the fuzzy soft simply open sets (F_{A_i})'s are not FSDS in (U,E,ψ).

Proposition 4.6:

If $F_A$is a FSFCS in a FSSBS , then F_A is a FSDS in (U,E,ψ).

Proof:

Let F_A be a FSFCS in (U,E,ψ). By prop 3.20 , there exist a FSSFCS G_B in (U,E,ψ). Such that $cl^{fs}(F_A) \geq G_B$.Then $cl^{fs}[cl^{fs}(F_A)] \geq cl^{fs}(G_B)$. Since (U,E,ψ) is a FSSBS by prop 4.1, for the FSSFCS G_B in (U,E,ψ), $cl^{fs}(G_B) = 1$. Then $cl^{fs}(F_A) \geq 1$. ie) $cl^{fs}(F_A) = 1$. Hence F_A is a FSDS in (U,E,ψ).

Proposition 4.7:

If H_C is a FSRS in a FSSBS , then $int^{fs}(H_E) = 0$ in (U,E,ψ).

Proof:

Let H_C be a FS residual set in (U,E,ψ). Then $1 - H_C$is a FSFCS in (U,E,ψ). since (U,E,ψ) is a FSSBS, by prop 4.6, $cl^{fs}(1-H_C) = 1$ in (U,E,ψ). Then $1 - int^{fs}(H_C) = 1$ and hence $int^{fs}(H_C) = 0$ in (U,E,ψ).

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On Fuzzy Soft Strongly Baire Spaces

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ABSTRACT. The aim of this paper is to introduce and study the concept of fuzzy soft weak Baire spaces are introduced, and their characterizations and properties are investigated in this work. Several examples are given to illustrate the concepts introduced in this paper.

1. INTRODUCTION

The concept of fuzzy set where launched by Lotfi A. Zadeh in 1965 [12]. C.L. Chang [2] used fuzzy sets and defined by fuzzy topological space also C.L. Chang found the way for the Massive extension of the countless fuzzy topolog-ical concepts. In classical topology, the concept of σ-boundary sets was defined by J. Martinez and W.W. Mc Govern [7]. The concept of fuzzy soft set fss, for short is introduced and studied [3–6, 8] a more generalized concept which is a combination of fuzzy set and fss.

In this paper the idea of fuzzy soft σ-boundary set used fuzzy soft topological spaces.

2. PRELIMINARIES

De inition 2.1. *[3] The fss $F_\phi \in FS(U, E)$ is said to be null fss and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fss 0 of U, where $0(x) = 0$ for all $x \in U$.*

Definition 2.2. *[3] Let $F_E \in FS(U,E)$ and $F_E(e) = \overline{1}$ all $e \in E$, where $\overline{1}(x) = 1$ for all $x \in U$. Then F_E is called absolute fss. It is denoted by $\overline{E}$.*

Definition 2.3. *[3] A fss F_A is said to be a fuzzy soft subset of a fss G_B over a common universe U if $A \subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e \in A$, i.e., if $\mu^e_{F_A}(x) \leq \mu^e_{G_B}(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F_A \breve{\subseteq} G_B$.*

Definition 2.4. *[3] Two fsss F_A and G_B over a common universe U are said to be fuzzy soft equal if F_A is a fuzzy soft subset of G_B and G_B is a fuzzy soft subset of F_A.*

Definition 2.5. *[3] The union of two fsss F_A and G_B over the common universe U is the fss H_C, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \breve{\bigvee} G_B$.*

Definition 2.6. *[3] Let F_A and G_B be two fsss. Then the intersection of F_A and G_B is a fss H_C, defined by $H(e) = \mu^e_{F_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \breve{\wedge} G_B$.*

Lemma 2.1. *[1] For a family $A = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\bigvee(cl(\lambda_\alpha)) \leq cl(\bigvee(\lambda_\alpha))$. In case A is a finite set, $\bigvee(cl(\lambda_\alpha)) = cl(\bigvee(\lambda_\alpha))$. Also $\bigvee(int(\lambda_\alpha)) \leq int(\bigvee(\lambda_\alpha))$.*

Definition 2.7. *[11] Let $F_A \in FS(U,E)$ be a fss. Then the complement of F_A, denoted by F^C_A, defined by*

$$F^C_A(e) = \begin{cases} \overline{1} - \mu^e_{F_A}, \text{ if } e \in A \\ \overline{1}, \text{ if } e \notin A \end{cases}$$

Definition 2.8. *[10] Let ψ be the collection of fsss over U. Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:*

- *(i) Φ, $\overline{E}$ belong to ψ.*
- *(ii) The union of any number of fsss in ψ belongs to ψ.*
- *(iii) The intersection of any two fsss ψ belongs to ψ.*

The triplet (U, E, ψ) is called a fuzzy soft topological space over U. The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

Definition 2.9. *[9] The union of all fuzzy soft open subsets of F_A over (U, E, ψ) is called the interior of F_A and is denoted by $int^{fs}(F_A)$.*

Theorem 2.1. *[9]* $int^{fs}(F_A \breve{\wedge} G_B) = int^{fs}(F_A) \breve{\wedge} int^{fs}(G_B)$.

Definition 2.10. *[9] Let $F_A \in FS(U, E)$ be a fss. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.*

Remark 2.1. *[9]*

(1) *For any fss F_A in a fuzzy soft topological space (U, E, ψ), it is easy to see that $((F_A))^c = int^{fs}(F_A^C)$ and $(int^{fs}(F_A))^c = cl^{fs}(F_A^C)$.*
(2) *For any fuzzy soft F_A subset of a fuzzy soft topological space (U, E, ψ), we define the fuzzy soft subspace topology. ψ on F_A by $K_D \in \psi_{F_A}$ if $K_D = F_A \breve{\wedge} G_B$ for some $G_B \in \psi$.*
(3) *For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of H_C in F_A by $int^{fs}_{F_A}(H_C)$ and $cl^{fs}_{F_A}(H_C)$, respectively.*

3. Fuzzy soft σ-boundary set

Definition 3.1. *A fss F_A in a FSTS (U, E, ψ) is called a fs σ-boundary set if $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$, where $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge (1 - F_{A_i})$ and (F_{A_i})'s are fuzzy soft regular open sets (fsro, for short) in (U, E, ψ).*

Example 1. *Let $U = \{x_1, x_2, x_3\}$. The fuzzy sets F_E, G_E, H_E are defined on U as follows. Let*

$$F_E = \begin{bmatrix} 0.6 & 0.3 & 0.3 \\ 0.4 & 0.4 & 0.3 \\ 0.4 & 0.3 & 0.5 \end{bmatrix}, \quad G_E = \begin{bmatrix} 0.4 & 0.5 & 0.6 \\ 0.5 & 0.6 & 0.7 \\ 0.5 & 0.5 & 0.6 \end{bmatrix}, \quad H_E = \begin{bmatrix} 0.4 & 0.6 & 0.5 \\ 0.5 & 0.5 & 0.6 \\ 0.7 & 0.7 & 0.8 \end{bmatrix}.$$

Then $\psi = \{0, F_E, G_E, H_E, 1\}$ is clearly fst on (U, E, ψ). The fs σ-boundary set in (U, E, ψ) are $G_{B_1} \bigvee G_{B_2} \bigvee G_{B_3} = 1 - F_{E_i}$ implies $1 - F_E$ is a fs σ-boundary set in (U, E, ψ).

Proposition 3.1. *If F_A is a fs σ-boundary set in a FSTS (U, E, ψ), then F_A is a fs F_σ-set in (U, E, ψ).*

Proof. Let F_A be a fs σ-boundary set in a FSTS (U, E, ψ). Then $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$ where $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge (1 - F_{A_i})$ and (F_{A_i})' s are fsro sets in (U, E, ψ). Since (F_{A_i})'s are fsro sets in (U, E, ψ). $(1 - F_{A_i})$ is a fuzzy soft regular closed set (fsrcs, for short) in (U, E, ψ). This implies that $(1 - F_{A_i})$ is a fsrcs in (U, E, ψ) and hence

$cl^{fs}(1-F_{A_i}) = (1-F_{A_i})$ in (U,E,ψ). Then $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge cl^{fs}(1-F_{A_i})$ is a fscs in (U,E,ψ). Thus $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$, where (G_{B_i})' s are fscs in (U,E,ψ), implies that F_A is a fs F_σ-set in (U,E,ψ). □

Proposition 3.2. *If $G_B = cl^{fs}(F_A) \bigwedge (1-F_A)$, where F_A is a fsro sets in a FSTS (U,E,ψ), then G_B is a fuzzy soft pre-closed set (fspcs, for short) in (U,E,ψ).*

Proof. Let F_A be a fsro sets in (U,E,ψ), then $int^{fs}[cl^{fs}(F_A)] = F_A$ in (U,E,ψ). Now, $G_B = cl^{fs}(F_A) \bigwedge (1-F_A) = cl^{fs}(int^{fs}[cl^{fs}(F_A)]) \bigwedge (1 - int^{fs}[cl^{fs}(F_A)]) = cl^{fs}(int^{fs}[cl^{fs}(F_A)]) \bigwedge cl^{fs}[int^{fs}(1-F_A)] \geq cl^{fs}[(int^{fs}[cl^{fs}(F_A)])]$ $= cl^{fs}int^{fs}[cl^{fs}(F_A) \bigwedge (1-F_A)] = cl^{fs}(G_B)$. Thus $cl^{fs}int^{fs}(G_B) \leq G_B$ in (U,E,ψ) implies that G_B is a fspcs in (U,E,ψ). □

Proposition 3.3. *If $G_B = cl^{fs}(F_A) \bigwedge (1-F_A)$, where F_A is a fsro sets in (U,E,ψ), then $int^{fs}(G_B) = F_A \bigwedge int^{fs}(1-F_A)$ in (U,E,ψ).*

Proof. Suppose that $G_B = Cl(F_A) \bigwedge (1-F_A)$. Then $int^{fs}(G_B) = int^{fs}cl^{fs}(F_A) \bigwedge (1-F_A) = int^{fs}cl^{fs}(F_A) \bigwedge int^{fs}(1-F_A) = int^{fs}cl^{fs}(F_A) \bigwedge [1 - cl^{fs}(F_A)] = F_A \bigwedge [1 - cl^{fs}(F_A)]$, Since F_A is a fsro sets in (U,E,ψ), $int^{fs}cl^{fs}(F_A) = F_A$. Then $int^{fs}(G_B) = F_A \bigwedge [1 - cl^{fs}(F_A)] = F_A \bigwedge [int^{fs}(1-F_A)]$ in (U,E,ψ). □

Proposition 3.4. *If $G_B = cl(F_A) \bigwedge (1-F_A)$, where F_A is a fsro sets in (U,E,ψ), then $cl^{fs}(G_B) \leq$ fsbd (F_A) where fsbd (F_A) is the fs boundary of F_A in (U,E,ψ).*

Proof. Suppose that $G_B = CL(F_A) \bigwedge (1-F_A)$, where F_A is a fsro sets in (U,E,ψ). Then $cl^{fs}(G_B) = cl^{fs}[cl^{fs}(F_A) \bigwedge (1-F_A)] \leq cl^{fs}cl^{fs}(F_A) \bigwedge (1-F_A) = cl^{fs}(F_A) \bigwedge cl^{fs}(1-F_A)$. This implies that $cl^{fs}(G_B) \leq [cl^{fs}(F_A) \bigwedge cl^{fs}(1-F_A)] =$ fsbd (F_A). Thus $cl^{fs}(G_B) \leq$ fsbd (F_A) in (U,E,ψ). □

Proposition 3.5. *If fsbd (F_A) is a fuzzy soft nowhere dense set (fsnds, for short), for a fsro set F_A in (U,E,ψ), then $G_B = CL^{fs}(F_A) \bigwedge (1-F_A)$ is a fsnds in (U,E,ψ).*

Proof. Let F_A be a fsro set in (U,E,ψ) such that $int^{fs}cl^{fs}[fsbd(F_A)] = 0$. Suppose that $G_B = CL^{fs}(F_A) \bigwedge (1-F_A)$ in (U,E,ψ). Then by proposition 3.4, $cl(G_B) \leq$ fsbd (F_A), in (U,E,ψ). Then $int^{fs}cl^{fs}[cl(G_B)] \leq int^{fs}cl^{fs}[fsbd(F_A)]$ in (U,E,ψ) and hence $int^{fs}cl^{fs}(G_B) \leq int^{fs}cl^{fs}[fsbd(F_A)]$ in (U,E,ψ). Since $int^{fs}cl^{fs}[fsbd(F_A)] = 0$, $int^{fs}cl^{fs}(G_B) \leq 0$ in (U,E,ψ). That is., $int^{fs}cl^{fs}(G_B) = 0$ in (U,E,ψ) and hence G_B is a fsnds in (U,E,ψ). □

Proposition 3.6. *If F_A is a fs σ-boundary set in a FSTS (U, E, ψ), then $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$, where (G_{B_i})'s are fspcs in (U, E, ψ).*

Proof. Let F_A be a fs σ-boundary set in (U, E, ψ). Then $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$, where $G_{B_i} = CL^{fs}(F_{A_i}) \bigwedge(1 - F_{A_i})$and (F_{A_i})'s are fsro set in (U, E, ψ). By proposition 3.2, (G_{B_i})'s are fpcs in (U, E, ψ), and hence $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$, where$(G_{B_i})$'s are fspcs in (U, E, ψ). □

Definition 3.2. *A fss F_A in a Fsts (U, E, ψ) is called a fsp F_σ-set if $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$, where G_{B_i}'s are fspcs in (U, E, ψ).*

Proposition 3.7. *If F_A is a fs σ-boundary set in a FSTS (U, E, ψ), then F_A is a fsp F_σ-set in (U, E, ψ).*

Proof. The proof follows from definition 3.2 and proposition 3.6. □

Definition 3.3. *A fs set H_E in a FSTS in (U, E, ψ) is called a fuzzy soft co-σ-boundary set, if $H_E = \bigwedge_{i=1}^{\infty}(H_{E_i})$ where $H_{E_i} = int^{fs}(1 - F_{A_i}) \bigvee F_{A_i}$ and (F_{A_i})'s are fso sets in (U, E, ψ).*

Proposition 3.8. *If H_E is a fs co-σ-boundary set in (U, E, ψ), then $1 - H_E$ is a fs σ-boundary set in (U, E, ψ).*

Proof. Let H_E be a fs co-σ-boundary set in (U, E, ψ). Then $H_E = \bigwedge_{i=1}^{\infty}(H_{E_i})$, where $H_{E_i} = int^{fs}(1 - F_{A_i}) \bigvee F_{A_i}$ and (F_{A_i})'s are fsro sets in (U, E, ψ). Now $1-H_E = 1-\bigwedge_{i=1}^{\infty}(H_{E_i}) = \bigvee_{i=1}^{\infty}(1-H_{E_i})$, in (U, E, ψ). Also $1-H_{E_i} = 1-[int^{fs}(1-F_{A_i}) \bigvee(F_{A_i})] = [1-int^{fs}(1-F_{A_i})] \bigwedge(1-F_{A_i}) = \{1-[1-cl^{fs}(F_{A_i})]\} \bigwedge(1-F_{A_i})$ and hence $1-H_{E_i} = cl^{fs}(F_{A_i}) \bigwedge(1-F_{A_i})$. Let $G_{B_i} = 1-H_{E_i}$, then $1-H_E = \bigvee_{i=1}^{\infty}(G_{B_i})$ where $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge(1 - F_{A_i})$ and (F_{A_i})'s are fsros in (U, E, ψ). Thus $1 - H_E$ is a fs σ-boundary set in (U, E, ψ). □

4. Fuzzy Soft Weak Baire Spaces

Definition 4.1. *A FSTS (U, E, ψ) is called a fswbs if $int^{fs}[\bigvee_{i=1}^{\infty}(G_{B_i})] = 0$,where $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge(1 - F_{A_i})$ and (F_{A_i})'s are fsro set in (U, E, ψ).*

Example 2. *Let $U = \{x_1, x_2, x_3\}$. The fuzzy sets F_E, G_E, H_E, are defined on U as follows. Let*

$$F_E = \begin{bmatrix} 0.6 & 0.8 & 0.7 \\ 0.5 & 0.3 & 0.7 \\ 0.5 & 0.3 & 0.4 \end{bmatrix} \quad G_E = \begin{bmatrix} 0.4 & 0.7 & 0.7 \\ 0.3 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.8 \end{bmatrix} \quad H_E = \begin{bmatrix} 0.6 & 0.5 & 0.7 \\ 0.6 & 0.4 & 0.6 \\ 0.5 & 0.3 & 0.4 \end{bmatrix}.$$

Then, $\Psi = \{0, F_E, G_E, H_E, 1\}$ is clearly fst on (U, E, Ψ). Now consider the fs σ-boundary set in (U, E, ψ) are $(1 - G_E \bigvee H_E)1 - F_E$. Now we prove

$$\begin{aligned} Int(\bigvee_{i=1}^{\infty} H_B) &= G_{B_1} \bigvee G_{B_2} \bigvee G_{B_3} \\ &= int(1 - G_E \bigvee H_E) \\ &= 0. \end{aligned}$$

Hence (U, E, ψ) is a fswbs.

Proposition 4.1. *Let (U, E, ψ) be a fsts. Then, the following are equivalent;*

- *(i) (U, E, ψ) is a fuzzy soft weak Baire space (fswbs, for short).*
- *(ii) $int^{fs}(F_A) = 0$, for each fs σ-boundary set F_A in (U, E, ψ).*
- *(iii) $cl^{fs}(H_E) = 1$, for each fs co-σ-boundary set H_E in (U, E, ψ).*

Proof.

$(i) \Rightarrow (ii)$ Let F_A be a fuzzy σ-boundary set in (U, E, ψ). Then $F_A = \bigvee_{i=1}^{\infty}(G_{B_i})$, where $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge (1 - F_{A_i})$ and (F_{A_i})'s are fsro set in (U, E, ψ). Then $int^{fs}(F_A) = int^{fs}[\bigvee_{i=1}^{\infty}(G_{B_i})] = 0$, since (U, E, ψ) is a fswbs. Thus, for the fs -boundary set F_A in (U, E, ψ), $int^{fs}(F_A) = 0$ in (U, E, ψ).

$(ii) \Rightarrow (iii)$ Let H_E be a fs co-σ-boundary set in (U, E, ψ), then by proposition 3.8, $1 - H_E$ is a fuzzy soft σ-boundary set in (U, E, ψ). Then, by hypothesis, $int^{fs}(1 - H_E) = 0$ in (U, E, ψ). Now $cl^{fs}(H_E) = 1 - [1 - cl^{fs}(H_E)] = 1 - [int^{fs}(1 - H_E)] = 1 - 0 = 1$. Thus $cl^{fs}(H_E) = 1$, for the fs co-σ-boundary set H_E in (U, E, ψ).

$(iii) \Rightarrow (i)$ Let H_E be a fs co-σ-boundary set in (U, E, ψ). Then $1 - H_E$ is a fs σ-boundary set in (U, E, ψ) and hence $1 - H_E = \bigvee_{i=1}^{\infty}(G_{B_i})$ where $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge (1 - F_{A_i})$ and (F_{A_i})'s are fsro sets in (U, E, ψ). Then $int^{fs}(1 - H_E) = int^{fs}[\bigvee_{i=1}^{\infty}(G_{B_i})]$. Now, by hypothesis $cl^{fs}(H_E) = 1$ in (U, E, ψ). Since $int^{fs}(1 - H_E) = 1 - cl^{fs}(H_E) = 1 - 1 = 0$, $int^{fs}[\bigvee_{i=1}^{\infty}(G_{B_i})] = 0$, where $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge (1 -$

F_{A_i}) and (F_{A_i})'s are fsro sets in (U, E, ψ). This implies that (U, E, ψ) is a fswbs. □

Proposition 4.2. *If (U, E, ψ) is a fswbs, then for any fsro set F_A in (U, E, ψ), $[F_A \bigwedge int^{fs}(1 - F_A)] = 0$, in (U, E, ψ).*

Proof. Let (U, E, ψ) be a fswbs. Then $int^{fs}[\bigvee_{i=1}^{\infty}(G_{B_i})] = 0$, where $G_{B_i} = cl^{fs}(F_{A_i}) \bigwedge (1 - F_{A_i})$ and (F_{A_i})'s are fuzzy soft regular open sets in (U, E, ψ). By lemma 2.1, $\bigvee_{i=1}^{\infty} int^{fs}(G_{B_i}) \leq int^{fs}[\bigvee_{i=1}^{\infty}(G_{B_i})]$ in (U, E, ψ). Then $\bigvee_{i=1}^{\infty} int^{fs}(G_{B_i}) \leq 0$ in (U, E, ψ). That is., $\bigvee_{i=1}^{\infty} int^{fs}(G_{B_i}) = 0$ in (U, E, ψ). This implies that $int^{fs}(G_{B_i}) = 0$ in (U, E, ψ). Then from proposition 3.3, $[F_{A_i} \bigwedge int^{fs}(1 - F_{A_i})] = 0$, in (U, E, ψ). Thus, if F_A is a fsro set in (U, E, ψ), then $F_A \bigwedge int^{fs}(1 - F_A) = 0$ in (U, E, ψ). □

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Soft semi-pre-Baire spaces on fuzzy soft topological spaces

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Abstract. The main aim of this paper is to initiate and explore the properties of fuzzy soft semi-pre-Baire spaces. We introduce and investigate fuzzy soft semi-pre-nowhere dense set, fuzzy soft semi-pre first category set, fuzzy soft semi-pre-second category set in fuzzy soft topological spaces.

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Keywords: Fuzzy soft semi-pre-nowhere dense set, fuzzy soft semi-pre-first category set, fuzzy soft semi-pre-second category set, fuzzy soft semi-pre-residual set, fuzzy soft semi-pre-Baire space.

1. Introduction

A number of real life problems in engineering, social and medical sciences, economics etc. involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. Some of these problems are essentially humanistic and thus subjective in nature (e.g. human understanding and vision systems), while others are objective, yet they are firmly embedded in an imprecise environment. The concept of fuzzy sets was first introduced by Zadeh [1] in the year 1965. This concept provides a natural foundation for treating mathematically the fuzzy soft phenomena which exist pervasively in our heal world and for new branches of fuzzy soft mathematics. Thenceforth, Chang [2] made tremendous growth of the numerous fuzzy soft topological concepts. Molodtsov [3] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. Later other authors like Maji et al. [4] further studied the theory of soft sets and to solve some decision making problems. The concept of fuzzy soft set is introduced and studied [5, 6, 7, 8, 9] as a more generalized concept which is a combination of fuzzy set and fuzzy soft set. Since then much attention has been paid to generalize the basic concept of fuzzy

topology in soft setting and this modern theory of FST has been developed. In recent years, FST has been found to be very useful in solving any practical problem. The aim of this paper is to introduce the concept of fuzzy soft semi-pre-nowhere dense sets, FSSPFC, FSSPSC, FSSPR and FSSPBS in FSTS. In section 3, we have presented a brief note on the preliminaries related to soft sets definitions around our characterizations. In section 4 deals with FSSPNDS and their characterizations are established in FSTS. In section 5, FSSPBS are introduced. Finally, we have some conclusion in concluding section 6.

2. Review of Literature

In this paper ,the major work for fuzzy soft semi-pre-nowhere dense sets are introduced and further countable union of fuzzy soft semi-pre-nowhere dense sets are zero are called fuzzy soft semi-pre-Baire spaces.

3. Preliminaries

Now we give some basic notions and results that are used in the sequel. In this work by using a FSTS we shall mean a non-empty set U together with a FST U (in the sense of Chang) and denote it by (U, E, ψ). The interior, closure and the complement of a fuzzy soft set F_A will be denoted by $int^{fs}(F_A)$, $cl^{fs}(F_A)$ and $1-F_A$ respectively.

Definition 3.1 ([6]). The fuzzy soft set $F_\phi \in FS(U, E)$ is said to be *null fuzzy soft set* and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fuzzy soft set $\overline{0}$ of U, where $\overline{0}(x) = 0$ for all $x \in U$.

Definition 3.2 ([6]). Let $F_E \in FS(U, E)$ and $F_E(e) = \overline{1}$ all $e \in E$, where $\overline{1}(x) = 1$ for all $x \in U$. Then F_E is called an *absolute fuzzy soft set*. It is denoted by $\overline{E}$.

Definition 3.3 ([6]). A fuzzy soft set F_A is said to be a *fuzzy soft subset* of a fuzzy soft set G_B over a common universe U, if $A \subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e \in A$, i.e., if $\mu^e_{F_A}(x) \leq \mu^e_{G_B}(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F_A \breve{\subseteq} G_B$.

Definition 3.4 ([6]). Two fuzzy soft sets F_A and G_B over a common universe U are said to be *fuzzy soft equal*, if F_A is a fuzzy soft subset of G_B and G_B is a fuzzy soft subset of F_A.

Definition 3.5 ([6]). The *union* of two fuzzy soft sets F_A and G_B over the common universe U is the fuzzy soft set H_C, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \breve{\bigvee} G_B$.

Definition 3.6 ([6]). Let F_A and G_B be two fuzzy soft sets. Then the *intersection* of F_A and G_B is a fuzzy soft set H_C, defined by $H(e) = \mu^e_{F_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \breve{\bigwedge} G_B$.

Definition 3.7 ([10]). Let $F_E \in FS(U, E)$ be a fuzzy soft set. Then the *complement* of F_A, denoted by F^c_A, defined by

$$F^c_A(e) = \begin{cases} \overline{1} - \mu^e_{F_A}, \text{ if } e \in A \\ \overline{1}, \text{ if } e \notin A \end{cases}$$

Definition 3.8 ([11]). Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a *fuzzy soft topology* on U, if ψ satisfies the following axioms:

(i) Φ, $\overline{E}$ belong to ψ,
(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ,
(iii) The intersection of any two fuzzy soft sets ψ belongs to ψ.

The triplet (U, E, ψ) is called a *fuzzy soft topological space* over U. The members of ψ are called *fuzzy soft open sets* in U and complements of them are called *fuzzy soft closed sets* in U.

Definition 3.9 ([11]). The union of all fuzzy soft open subsets of F_A over (U, E, ψ) is called the *interior* of F_A and is denoted by $int^{fs}(F_A)$.

Definition 3.10 ([11]). Let $F_A \in FS(U, E)$ be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the *closure* of F_A and is denoted by $cl^{fs}(F_A)$.

Remark 3.11. [11]

(1) For any fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ), it is easy to see that $(cl^{fs}(F_A))^c = int^{fs}(F_A^c)$ and $(int^{fs}(F_A))^c = cl^{fs}(F_A^c)$.
(2) For any fuzzy soft F_A subset of a fuzzy soft topological space (U, E, ψ), we define the fuzzy soft subspace topology ψ on F_A by $K_D \in \psi_{F_A}$ if $K_D = F_A \breve{\wedge} G_B$ for some $G_B \in \psi$.
(3) For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of H_C in F_A by $int^{fs}_{F_A}(H_C)$ and $cl^{fs}_{F_A}(H_C)$, respectively.

Definition 3.12 ([12]). A fuzzy soft set F_A in a FSTS (U, E, ψ) is called a *fuzzy soft neighborhood* (briefly, FS nbd) *set*, if there exists no non-zero fuzzy soft open set G_B in (U, E, ψ) such that $G_B < cl^{fs}(F_A)$. i.e., $int^{fs}cl^{fs}(F_A) = 0$.

Definition 3.13 ([12]). A fuzzy soft set F_A in a FSTS (U, E, ψ) is called *fuzzy soft dense*, if there exists no fuzzy soft closed set G_B in (U, E, ψ) such that $F_A < G_B < 1$. i.e., $cl^{fs}(F_A) = 1$.

Definition 3.14 ([12]). Let (U, E, ψ) be a fuzzy soft topology. A fuzzy soft set F_A in (U, E, ψ) is called *fuzzy soft first category*, if $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft nowhere dense sets in (U, E, ψ). Any other fuzzy soft set in (U, E, ψ) is said to be of fuzzy soft second category.

Definition 3.15 ([13]). Let (X, T, E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$. If $f_A | Fcl(Fint(f_A))$, then f_A is called fuzzy semi open soft set. We denote the set of all fuzzy semi open soft sets by $FSOS(X, T, E)$, or $FSOS(X)$ and the set of all fuzzy semi closed soft sets by $FSCS(X, T, E)$, or $FSCS(X)$.

Theorem 3.16 ([13]). *Let (X, T, E) be a fuzzy soft topological space and $f_A \in FSOS(X)$. Then*

(1) *arbitrary fuzzy soft union of fuzzy semi open soft set is fuzzy semi open soft,*
(2) *arbitrary fuzzy soft intersection of fuzzy semi closed soft set is fuzzy semi closed soft.*

Theorem 3.17 ([13]). *Let (X, T, E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then*

(1) *$f_A \in FSOS(X)$ if and only if $Fcl(f_A) = Fcl(Fint(f_A))$,*
(2) *if $g_B \in T$, then $g_B Fcl(f_A) | Fcl(g_B g_B)$.*

Theorem 3.18 ([13]). *Let (X, T, E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then*

(1) *$f_A \in FSOS(X)$ if and only if there exists $g_B \in T$ such that $g_B \hat{o} f_A \hat{o} Fcl(g_B)$,*
(2) *if $f_A FSOS(X)$ and $f_A \hat{o} h_D \hat{o} Fcl(f_A)$, then $h_D \in FSOS(X)$.*

Definition 3.19 ([13]). Let (X, T, E) be a fuzzy soft topological space, $f_A \in FSS(X)_E$ and $f_e \in FSS(X)_E$. Then

(i) f_e is called *fuzzy semi interior soft point* of f_A, if there exists $g_B \in FSOS(X)$ such that $f_e \in g_B \hat{o} f_A$. The set of all fuzzy semi interior soft points of f_A is called the fuzzy semi soft interior of f_A and is denoted by $FSint(f_A)$. Consequently, $FSint(f_A) = \{g_B : g_B | f_A, g_B \in FSOS(X)\}$.
(ii) f_e is called *fuzzy semi cluster soft point* of f_A, if $f_A h_D \notin \hat{o}_E, \forall h_D \in FSOS(X)$. The set of all fuzzy semi cluster soft points of f_A is called fuzzy semi soft closure of f_A and denoted by $FScl(f_A)$. Consequently, $FScl(f_A) = \{h_D : h_D \in FSCS(X), f_A\ h_D\}$.

Theorem 3.20 ([13]). *Let (X, T, E) be fuzzy soft topological space and f_A, $g_B \in FSS(X)_E$. Then the following properties are satisfied for the fuzzy semi interior operator, denoted by $FSint$.*

(1) *$FSint(\tilde{1}_E) = \tilde{1}_E$ and $FSint(\tilde{0}_E) = \tilde{0}_E$.*
(2) *$FSint(f_A) \hat{o} f_A$.*

Definition 3.21 ([14]). A fuzzy set λ in a fuzzy topological space (X, T) is called a *fuzzy F_σ-set* in (X, T), if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(1 - \lambda_i) \in T$ for $i \in I$.

4. Fuzzy soft semi-pre-nowhere dense set

Motivated by the concept of fuzzy soft semi-pre-nowhere dense sets on FSTS.

Definition 4.1. A fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ) is called a *fuzzy soft semi-pre- nowhere dense set*, if there exists no non-zero fuzzy soft semi-pre-open set G_B in (U, E, ψ) such that $G_B < spcl^{fs}(F_A)$. That is,

$$spint^{fs} spcl^{fs}(F_A) = 0.$$

Example 4.2. Let $U = \{x_1, x_2, x_3\}$. Consider F_E, G_E, H_E are the fuzzy sets on U defined as follows:

$$F_E = \begin{bmatrix} 7 & 7 & 4 \\ 6 & 5 & 7 \\ 7 & 5 & 6 \end{bmatrix}, G_E = \begin{bmatrix} 10 & 5 & 6 \\ 7 & 6 & 7 \\ 8 & 5 & 8 \end{bmatrix}, H_E = \begin{bmatrix} 7 & 7 & 6 \\ 8 & 72 & 62 \\ 56 & 82 & 62 \end{bmatrix}.$$

Then $\psi = \{0, F_E, G_E, H_E, 1\}$ is clearly fuzzy soft topology on (U, E, ψ). Now consider the fuzzy soft semi set in (U, E, ψ) are $int^{fs} cl^{fs}(G_E) \leq G_E \leq (1 - G_E)$. $int^{fs} cl^{fs}(H_E) \leq H_E \leq (1 - F_E \wedge G_E)$. Then $int^{fs} cl^{fs}(H_E) = int^{fs}(1 - F_E \wedge G_E) = 0$. Thus (U, E, ψ) is a fuzzy soft semi-pre-nowhere dense sets.

Proposition 4.3. *The complement of a fuzzy soft semi-pre-nowhere dense set in a FSTS (U, E, ψ) need not be a fuzzy soft semi-pre-nowhere dense set.*

Proof. In Example 4.2, $(1 - F_E)$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ), where as $F_E = 1 - (1 - F_E)$ is not a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.4. *If F_A and G_B are fuzzy soft semi-pre-nwd sets in a FSTS (U, E, ψ), then $F_A \vee G_B$ need not be a fuzzy soft semi-pre-nwd set in (U, E, ψ).*

Proof. For in Example 4.2, $1 - F_E$, $1 - F_A$ are fuzzy soft semi-pre-nwd sets in (U, E, ψ). But $(1 - F_E) \vee (1 - G_E) = H_E$ implies that $spint^{fs}spcl^{fs}(H_E) \neq 0$. Then union of fuzzy soft semi-pre-nowhere dense sets need not be fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.5. *If the fuzzy soft sets F_A and G_B are fuzzy soft semi-pre-nwd sets in a FSTS (U, E, ψ), then $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let the fuzzy soft sets F_A and G_B be fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now $spint^{fs}spcl^{fs}(F_A \wedge G_B) \leq spint^{fs}spcl^{fs}(F_A) \wedge spint^{fs}spcl^{fs}(G_B) \leq 0 \wedge 0 = 0$ [since $spint^{fs}spcl^{fs}(F_A) = 0$ and $spint^{fs}spcl^{fs}(G_B) = 0$]. That is, $spint^{fs}spcl^{fs}(F_A \wedge G_B) = 0$. Then $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.6. *If F_A is a fuzzy soft semi-pre-nowhere dense set in a FSTS (U, E, ψ), then $spint^{fs}(F_A) = 0$.*

Proof. Let F_A be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then we have $spint^{fs}spcl^{fs}(F_A) = 0$. Now, $F_A \leq spcl^{fs}(F_A)$. Thus we have

$$spint^{fs}(F_A) \leq spint^{fs}spcl^{fs}(F_A) = 0.$$

So $spint^{fs}(F_A) = 0$. □

Proposition 4.7. *If F_A is a fuzzy soft semi-pre-nowhere dense set and G_B is any fuzzy soft set in a FSTS (U, E, ψ), then $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy softsemi-pre-nowhere dense set in (U, E, ψ). Then we get

$$spint^{fs}spcl^{fs}(F_A) = 0.$$

On the other hand,

$$\begin{aligned} spint^{fs}spcl^{fs}(F_A \wedge G_B) &\leq spint^{fs}spcl^{fs}(F_A) \wedge spint^{fs}spcl^{fs}(G_B) \\ &\leq 0 \wedge spint^{fs}spcl^{fs}(G_B) = 0. \end{aligned}$$

Thus $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Definition 4.8. A fuzzy soft set F_A in a FSTS (U, E, ψ) is called *fuzzy soft Semi-pre-dense*, if there exists no fuzzy soft semi-pre- closed set G_B in (U, E, ψ) such that $F_A < G_B < 1$. That is, $spcl^{fs}(F_A) = 1$.

Proposition 4.9. *If F_A is a fuzzy soft semi-pre-dense and fuzzy soft semi-pre-open set in a fuzzy soft topological space (U, E, ψ) and if $G_B \leq 1 - F_A$, then G_B is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-dense set in (U, E, ψ). Then $spcl^{fs}(F_A) = 1$ and $spint^{fs}(F_A) = F_A$. Now $G_B \leq 1 - F_A$ implies that $spcl^{fs}(G_B) \leq spcl^{fs}(1 - F_A)$. Thus $spcl^{fs}(G_B) \leq 1 - spint^{fs}(F_A) = 1 - F_A$. So $spcl^{fs}(G_B) \leq (1 - F_A)$, which implies that $spint^{fs}spcl^{fs}(G_B) \leq spint^{fs}(1 - F_A) = 1 - spcl^{fs}(F_A) = 1 - 1 = 0$. That is, $spint^{fs}spcl^{fs}(G_B) = 0$. Hence G_B is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.10. *If F_A is a fuzzy soft semi-pre-nowhere dense set in a FSTS (U, E, ψ). Then $1 - F_A$ is a fuzzy soft semi-pre-dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then we get

$$spint^{fs}spcl^{fs}(F_A) = 0.$$

Now $F_A \leq spcl^{fs}(F_A)$ implies that $spint^{fs}(F_A) \leq spint^{fs}spcl^{fs}(F_A) = 0$. Thus $spint^{fs}(F_A) = 0$ and $spcl^{fs}(1 - F_A) = 1 - spint^{fs}(F_A) = 1 - 0 = 1$. So $1 - F_A$ is a fuzzy soft semi-pre-dense set in (U, E, ψ). □

Proposition 4.11. *If F_A is a fuzzy soft semi-pre-nowhere dense set in a FSTS (U, E, ψ), then $spcl^{fs}(F_A)$ is also a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-nowhere set in (U, E, ψ). Then we have

$$spint^{fs}spcl^{fs}(F_A) = 0.$$

Now $spcl^{fs}spcl^{fs}(F_A) = spcl^{fs}(F_A)$. Thus we get

$$spint^{fs}spcl^{fs}(spcl^{fs}(F_A)) = spint^{fs}spcl^{fs}(F_A) = 0.$$

So $spcl^{fs}(F_A)$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.12. *If F_A is a fuzzy soft semi-pre-nowhere dense set in a FSTS (U, E, ψ), then $1 - spcl^{fs}(F_A)$ is a fuzzy soft semi-pre-dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then by proposition 4.11, $spcl^{fs}(F_A)$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Also by proposition 4.11, $1 - spcl^{fs}(F_A)$ is a fuzzy soft semi-pre-dense set in (U, E, ψ). □

Proposition 4.13. *Let F_A be a fuzzy soft semi-pre-dense set in a FSTS (U, E, ψ). If G_B is any fuzzy soft set in (U, E, ψ), then G_B is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ) if and only if $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let G_B be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then we have

$$spint^{fs}spcl^{fs}(G_B) = 0.$$

On the other hand,

$$\begin{aligned} spint^{fs}spcl^{fs}(F_A \wedge G_B) &\leq spint^{fs}spcl^{fs}(F_A) \wedge spint^{fs}spcl^{fs}(G_B) \\ &\leq spint^{fs}spcl^{fs}(F_A) \wedge 0 = 0. \end{aligned}$$

That is, $spint^{fs}spcl^{fs}(F_A \wedge G_B) = 0$. Thus $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).

Conversely, let $F_A \wedge G_B$ be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then $spint^{fs}spcl^{fs}(F_A \wedge G_B) = 0$. Thus we get

$$spint^{fs}spcl^{fs}(F_A) \wedge spint^{fs}spcl^{fs}(G_B) = 0.$$

Since F_A is a fuzzy softsemi-pre-dense set in (U, E, ψ), $spcl^{fs}(F_A) = 1$. So we get

$$spint^{fs}(1) \wedge spint^{fs}spcl^{fs}(G_B) = 0.$$

That is, $(1) \wedge spint^{fs}spcl^{fs}(G_B) = 0$. Hence we have

$$spint^{fs}spcl^{fs}(G_B) = 0.$$

Therefore G_B is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Definition 4.14. Let (U, E, ψ) be a FSTS. A fuzzy soft set F_A in (U, E, ψ) is called *fuzzy soft semi-pre-first category*, if $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Any other fuzzy soft set in (U, E, ψ) is said to be of fuzzy soft semi-pre-second category.

Definition 4.15. Let F_A be a fuzzy soft semi-pre-first category set in a FSTS (U, E, ψ). Then $1 - F_A$ is called a *fuzzy soft semi-pre-residual set* in (U, E, ψ).

Proposition 4.16. *If F_A is a fuzzy soft semi-pre-first category set in a fuzzy soft topological space (U, E, ψ), then $1 - F_A = \wedge_{i=1}^{\infty}(F_{A_i})$, where $spcl^{fs}(G_{B_i}) = 1$.*

Proof. Let F_A be a fuzzy soft semi-pre-first category set in (U, E, ψ). Then we have $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now $1-F_A = \wedge_{i=1}^{\infty}(F_{A_i})$. Let $G_{B_i} = 1-(F_{A_i})$. Then $1-F_A = \wedge_{i=1}^{\infty}(F_{A_i})$. Since (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ), by proposition 4.13, $(1 - F_{A_i})$'s are fuzzy soft semi-pre-dense set in (U, E, ψ). Thus we get

$$spcl^{fs}(G_{B_i}) = spcl^{fs}(1 - (F_{A_i})) = 1.$$

So $1 - F_A = \wedge_{i=1}^{\infty}(F_{A_i})$. Where $spcl^{fs}(G_{B_i}) = 1$. □

Proposition 4.17. *If F_A be a fuzzy soft semi-pre-closed set in a fuzzy soft topological space (U, E, ψ) and if $spint^{fs}(F_A) = 0$, then F_A is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-closed set in (U, E, ψ). Then $spcl^{fs}(F_A) = F_A$. Now $spint^{fs}spcl^{fs}(F_A) = spcl^{fs}F_A$ and $spint^{fs}(F_A) = 0$. Thus we have

$$spint^{fs}spcl^{fs}(F_A) = 0.$$

So F_A is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Definition 4.18. A Fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ) is called a *fuzzy soft semi-pre-F_σ-set* in (U, E, ψ), if $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-closed sets in (U, E, ψ).

5. Fuzzy soft semi-pre-Baire spaces

Motivated by the concept of fuzzy soft semi-pre-Baire spaces and their characterizations on FSTS.

Definition 5.1. Let (U, E, ψ) be a FSTS. Then (U, E, ψ) is called a *fuzzy soft semi-pre-Baire space*, if $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).

Example 5.2. Let $F_E = \begin{bmatrix} .7 & .7 & .6 \\ .8 & .72 & .62 \\ .44 & .82 & .62 \end{bmatrix}$, $G_E = \begin{bmatrix} .7 & .7 & .4 \\ .6 & .5 & .7 \\ .7 & .55 & .6 \end{bmatrix}$, $H_E = \begin{bmatrix} .10 & .5 & .6 \\ .7 & .6 & .7 \\ .8 & .5 & .8 \end{bmatrix}$.
Now consider the fuzzy soft sets. Then we have

$$int^{fs}cl^{fs}(F_E) \leq F_E \leq 1 - (F_E \wedge G_E) int^{fs}cl^{fs}(F_E) = int^{fs}(1 - F_E \wedge G_E) = 0.$$

Thus $\vee_{i=1}^{\infty} int^{fs}cl^{fs}(F_E \vee G_E) = 0$ is a fuzzy soft semi-pre-Baire space. So we get

$$spint^{fs}cl^{fs}(F_E \vee G_E) = spint^{fs}(1 - F_E \vee G_E) = 0.$$

Hence (U, E, ψ) is a fuzzy soft semi-pre-Baire space.

Proposition 5.3. *If $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$ and (F_{A_i})'s are fuzzy soft semi-pre-closed sets in a FSTS (U, E, ψ), then (U, E, ψ) is a fuzzy soft semi-pre-Baire space.*

Proof. Let (F_{A_i})'s be a fuzzy soft semi-pre-closed set in (U, E, ψ). Since

$$spint^{fs}(F_{A_i}) = 0,$$

(F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Then we have $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Thus (U, E, ψ) is a fuzzy soft semi-pre-Baire space. □

Proposition 5.4. *If $spcl^{fs}[\wedge_{i=1}^{\infty}(F_{A_i})] = 1$, where (F_{A_i})'s are fuzzy soft semi-pre-dense and fuzzy soft semi-pre-open sets in a FSTS (U, E, ψ). Then (U, E, ψ) is a fuzzy soft semi-pre-Baire space.*

Proof. Now $spcl^{fs}[\wedge_{i=1}^{\infty}(F_{A_i})] = 1$. This implies that $1 - spcl^{fs}[\wedge_{i=1}^{\infty}(F_{A_i})] = 0$. Then we have $spint^{fs}[1 - \wedge_{i=1}^{\infty}(F_{A_i})] = 0$. Thus $spint^{fs}[\vee_{i=1}^{\infty}(1 - F_{A_i})] = 0$. Since (F_{A_i})'s are fuzzy soft semi-pre-dense sets in (U, E, ψ), $spcl^{fs}(F_{A_i}) = 1$ and $spint^{fs}(1 - F_{A_i}) = 1 - spcl^{fs}(F_{A_i}) = 1 - 1 = 0$. So we have

$$spint^{fs}[\vee_{i=1}^{\infty}(1 - F_{A_i})] = 0,$$

where $spint^{fs}(1 - F_{A_i}) = 0$ and $(1 - F_{A_I})$'s are fuzzy soft semi-pre-closed sets in (U, E, ψ). Hence by proposition 5.3, (U, E, ψ) is a fuzzy soft semi-pre-Baire space. □

Proposition 5.5. *Let (U, E, ψ) be a FSTS. Then the following are equivalent:*

(1). *(U, E, ψ) is a fuzzy soft semi-pre-Baire space,*
(2). *$spint^{fs}(F_A) = 0$, for every fuzzy soft semi-pre-first category set F_A in (U, E, ψ),*
(3). *$spcl^{fs}(G_B) = 1$, for every fuzzy soft semi-pre residual set G_B in (U, E, ψ).*

Proof. (1) $\Rightarrow$ (2): Let F_A be a fuzzy soft semi-pre-first category set in (U, E, ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now $spint^{fs}(F_A) = spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$ ((U, E, ψ) is a fuzzy soft semi-pre-Baire space). Thus $spint^{fs}(F_A) = 0$.

(2) $\Rightarrow$ (3): Let G_B be a fuzzy soft semi-pre-residual set in (U, E, ψ). Then $1 - G_B$ is a fuzzy soft semi-pre-first category set in (U, E, ψ). Thus by hypothesis, we get

$$spint^{fs}(1 - G_B) = 0,\ 1 - spcl^{fs}(G_B) = 0.$$

So we have $spcl^{fs}(G_B) = 1$.

(3) $\Rightarrow$ (1): Let F_A be a fuzzy soft semi-pre-first category set in (U, E, ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now $1 - F_A$ is a fuzzy soft semi-pre-residual set in (U, E, ψ). Since F_A is a fuzzy soft semi-pre-first category set in (U, E, ψ). By hypothesis, we have $spcl^{fs}(1 - F_A) = 1$. Thus $1 - spint^{fs}(F_A) = 1$, $spint^{fs}(F_A) = 0$. So $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Hence (U, E, ψ) is a fuzzy soft semi-pre-Baire space. □

Proposition 5.6. *IF a FSTS (U, E, ψ) is a fuzzy soft semi-pre-Baire space, then (U, E, ψ) is a fuzzy soft semi-pre-second category space.*

Proof. Let (U, E, ψ) be a fuzzy soft semi-pre-Baire space. Then $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Then $\vee_{i=1}^{\infty}(F_{A_i}) \neq 1_X$ (suppose $\vee_{i=1}^{\infty}(F_{A_i}) = 1_X$ $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = spint^{fs}(1_X)$, implies $0 = 1$, a contradiction. Thus (U, E, ψ) is a fuzzy soft semi-pre-second category space. □

Remark 5.7. The converse of the above proposition need not be true. A fuzzy soft semi-pre-second category space need not be fuzzy soft semi-pre-Baire space.

In Example 5.2, the fuzzy soft sets $1 - F_A$, $1 - G_B$, $1 - F_A \vee G_B$, $1 - F_E$ are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now

$$(1 - F_A) \vee (1 - G_B) \vee (1 - F_A \vee G_B) \vee (1 - F_E) = 1 - F_A \wedge G_B \neq 1_X$$

and $int(1 - F_A \wedge G_B) \neq 0$. Then the fuzzy soft topological space (U, E, ψ) is a fuzzy soft semi-pre-second category space but not a fuzzy soft semi-pre-Baire space.

Proposition 5.8. *If a FSTS (U, E, ψ) is a fuzzy soft semi-pre-Baire space, then no non-zero fuzzy soft semi-pre-open set in (U, E, ψ) is a fuzzy soft semi-pre-first category set in (U, E, ψ).*

Proof. Suppose that F_A is a non-zero fuzzy soft semi-pre-open set in (U, E, ψ) such that $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Then we have $spint^{fs}(F_A) = spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})]$. Since F_A is a non-zero fuzzy soft semi-pre-open set in (U, E, ψ), we have $spint^{fs}(F_A) = F_A$. Thus $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = F_A \neq 0$. But this is a contradiction to (U, E, ψ) being a fuzzy soft semi-pre-Baire space in which $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). So we must have $F_A \neq \vee_{i=1}^{\infty}(F_{A_i})$. Hence no non-zero fuzzy soft semi-pre-open set in (U, E, ψ) is a fuzzy soft semi-pre-first category set in (U, E, ψ). □

6. Conclusions

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [3] and easily applied to many problems having uncertainties from social life. In this paper, we gave the definition of the FSTS and applying this basics for characterizations and properties in soft semi pre Baire spaces with examples .Hence this paper may be helpful for finding a base to researches who want to work in the field of FSSPBS..

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On Fuzzy Supra Baire Spaces

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ABSTRACT: The idea of this paper is to introduce the concepts of fuzzy supra Baire spaces by means of fuzzy supra nowhere dense set in fuzzy supra topological spaces with suitable examples to illustrate the concepts. Several characterizations of fuzzy supra Baire spaces are also studied.

KEYWORDS: Fuzzy supra dense, Fuzzy supra F_σ - set, Fuzzy supra-G_δ set, Fuzzy supra first category, Fuzzy supra second category, Fuzzy Supra residual set.

1. INTRODUCTION

Zadeh [10] was first introduced in 1965 the concept of fuzzy sets and fuzzy set operations in his classical paper. In 1968, the theory of fuzzy topological spaces was introduced and developed by Chang [3]. In 1977, Haworth and McCoy [4], introduced and studied the concept of Baire spaces in topological spaces. In 1987, AbdEl-monsef et at [1] introduced and defined the basic notions of fuzzy supra topological spaces. The idea of this paper is to introduce the concepts of fuzzy supra Baire space in fuzzy supra topological spaces with examples by using fuzzy supra nowhere dense set. The several characterizations of fuzzy supra Baire spaces are obtained.

2. Preliminaries

Definition 2.1[2]

A collection δ* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied:

1) **0** and **1** belongs to δ*.

2) $g_\chi \in \delta^*$ for each $\chi \in \Lambda$ implies $(v_{\chi \in \Lambda} g_\chi) \in \delta^*$.

The pair (X, δ*) is called a fuzzy supra topological space. The elements of δ* are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition 2.2 [5].

The closure and interior of a fuzzy set $f \in I^X$ are defined respectively as

$cl^*(g) = \wedge \{ h/ h$ is a fuzzy closed set in X and $g \le h \}$

$int^*(g) = v \{ h / h$ is a fuzzy open set in X and $h \le g \}$

Clearly $cl^*(g)$ is the smallest fuzzy closed set containing g and $int^*(g)$ is the largest fuzzy open set contained in g.

Lemma 2.3 [5]

For a fuzzy set λ in a fuzzy supra topological space X,

(i) $1 - int^*(\chi) = cl^*(1-\chi)$,

(ii) $1 - cl^*(\chi) = int^*(1-\chi)$.

Definition 2.4 [6]

A fuzzy supra open set χ in FSTS(X, T*) is called fuzzy supra F_σ set in (X, T*) if $\chi = v_{i=1}^{\infty} (\chi_i)$, where $1-\chi_i \in T^*$ for $i \in I$,

Definition 2.5 [6]

A fuzzy supra open set χ in FSTS (X, T*) is called fuzzy supra G_σ set in (X, T*) if $\chi = \wedge_{i=1}^{\infty} (\chi_i)$, where $\chi_i \in T^*$ for $i \in I$.

Definition 2.6 [6]

A fuzzy set χ in a FSTS (X,T*) is called a FS dense if there exists no FS closed set β in (X,T*) such that $\chi < \beta < 1$. That is, $cl^*(\chi)=1$, in (X,T*).

3 . FUZZY SUPRA NOWHEREDENSE SET

Definition 3.1 [6]

A fuzzy set χ in FSTS (X,T*) is called a fuzzy supra nowhere dense set if there exists no non-zero fuzzy supra open set μ in (X,T*) such that $\mu < cl^*(\chi)$. That is, $int^* cl^*(\chi)=0$, in (X,T*).

Proposition 3.1

If χ is a fuzzy supra closed set in (X,T*),with $int^*(\chi)=0$then χ is a fuzzy supra nowhere dense set in (X,T*)

Proof

Let χ is a fuzzy supra closed set in (X,T*) then $cl^*(\chi) = \chi$,Now $int^* cl^* (\chi) = int^*(\chi)=0$ and hence χ is a fuzzy supra nowhere dense set in (X,T*) .

Proposition 3.2

If the fuzzy sets χ and μ are fuzzy supra nowhere dense sets in a fuzzy supra topological space (X,T*). Then $\chi \wedge \mu$ is a fuzzy supra nowhere dense set in (X,T*).

Proof

Let the fuzzy sets χ and $\mu$are fuzzy supra nowhere dense sets in (X,T*). Now $int^*cl^*(\chi \wedge \mu) \leq int^*cl^*(\chi) \wedge int^*cl^*(\mu) \leq 0 \wedge 0$ (since $int^*cl^*(\chi)=0$ and $int^*cl^*(\mu)=0$). That is, $int^*cl^*(\chi \wedge \mu)=0$. Hence $(\chi \wedge \mu)$ is a fuzzy supra nowhere dense set in(X,T*).

Proposition 3.3

If χ is a fuzzy supra nowhere dense set is a fuzzy supra topological space (X,T*). Then $int^*cl^*(\chi)=0$.

Proof

Let χ be a fuzzy supra nowhere dense set in (X,T*) .Then we have $int^*cl^*(\chi)=0$Now$cl^*(\chi)=\chi$, since χ is fuzzy supra closed set in (X,T*) this implies that $int^*cl^*(\chi)= int^*(\chi)=0$. Hence $int^*cl^*(\chi) =0$.

Proposition 3.4

If χ is a fuzzy supra nowhere dense set and $\mu$is any fuzzy supra set is a fuzzy supra topological space (X,T*) then $(\chi \wedge \mu)$is a fuzzy supra nowhere dense set in (X,T*).

Proof

Let χ be a fuzzy supra nowhere dense set in (X,T*).Then, $int^*cl^*(\chi)=0$.Now $int^* cl^* (\chi \wedge \mu) \leq int^* cl^* (\chi) \wedge int^* cl^* (\mu) \leq 0 \wedge int^* cl^* (\mu)=0$that is, $int^* cl^* (\chi \wedge \mu)=0$.Hence $(\chi \wedge \mu)$is a fuzzy supra nowhere dense set in (X,T*).

Proposition 3.5

If χ is a fuzzy supra dense and fuzzy supra open set in a fuzzy supra topological space (X,T*) and if $\chi \leq 1- \mu$, then $\mu$is a fuzzy supra nowhere dense set in (X,T*).

Proof

Let χ be a fuzzy supra dense set in (X,T*) then we have $cl^* (\chi) =1$ and $int^* (\chi) = \chi$.Now $\mu \leq 1- \chi$,this implies that $cl^*(\mu) \leq cl^*(1- \chi)$ .Then$cl^*(\mu) \leq 1-int^*(\chi)= 1- \chi$which implies that $int^*cl^*(\mu) \leq int^*(1- \chi)=1-cl^*(\chi)=1 - 1=0$.That is $int^*cl^*(\mu) =0$.Hence μ is a fuzzy supra nowhere dense set in (X,T*).

Proposition 3.6

If χ is a fuzzy supra nowhere dense set in a fuzzy supra topological space (X,T*), then $1- \chi$ is a fuzzy supra dense set in (X,T*).

Proof

Let χ be a fuzzy supra nowhere dense set in (X,T*).Then,$int^* cl^* (\chi)=0$.Now $\chi \leq cl^* (\chi)$ implies that $int^* (\chi) \leq int^* cl^* (\chi)= 0$. Then $int^* (\chi) = 0$. Now$cl^* (1- \chi)=1- int^* (\chi)=1 - 0=1$, then $cl^* (1- \chi) =1$. Hence $1- \chi$is a fuzzy supra dense set in (X,T*).

Proposition3.7

If χ is a fuzzy supra nowhere dense set in a fuzzy supra topological space (X,T*),then $cl^*(\chi)$ is also a fuzzy supra nowhere dense set in (X,T*).

Proof

Let χ be a fuzzy supra nowhere dense set in (X,T*).Then, $int^* cl^* (\chi) =0$. Now $cl^*cl^*(\chi)=cl^*(\chi)$. Now $int^* cl^*(cl^* (\chi))= int^* cl^* (\chi)=0$. Implies that $int^*cl^*(cl^* (\chi))=0$. Therefore $cl^*(\chi)$ is also a fuzzy supra nowhere dense set in (X,T*).

Proposition 3.8

If χ be a fuzzy supra closed set in a fuzzy supra topological space (X,T*) and if $int^*(\chi)=0$,then $\chi$is a fuzzy supra nowhere dense set in (X,T*).

Proof

Let χ be a fuzzy supra closed set in (X,T*). Then we have $cl^* (\chi)= \chi$. Now $int^* cl^* (\chi)= int^* (\chi)$ and $int^* (\chi)=0$ implies that $int^* cl^* (\chi)=0$.Hence $\chi$is a fuzzy supra nowhere dense set in (X,T*).

Definition 3.2

A fuzzy set χ in a FSTS (X,T*) is called a FS first category set if $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)'s are FS nowhere dense set in (X,T*). Any other fuzzy set in (X,T*) is said to be FS second category.

Definition 3.3

A Fuzzy supra topological space (X, T*) is called a Fuzzy supra first category if $\vee_{i=1}^{\infty}(\chi_i)=1$, where (χ_i)'s are Fuzzy supra nowhere dense set in (X, T*). A topological space which is not of Fuzzy supra first category, is said to be Fuzzy supra second category.

Definition 3.4

Let χ a fuzzy supra first category set in (X, T*), Then 1- χ is called a fuzzy supra residual set in (X, T*).

Proposition 3.9

If χ is a fuzzy supra first category set in a fuzzy supra topological space (X,T*),then $1- \chi = \wedge_{i=1}^{\infty}(\chi_i)$,where $cl^*(\mu_i) =1$.

Proof

Let χ be a fuzzy supra first category set in (X,T*). Then we have $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where χ_i's are fuzzy supra nowhere dense sets in (X,T*).Now $1-\chi = 1 - \vee_{i=1}^{\infty}(\chi_i) = \wedge_{i=1}^{\infty}(\chi_i)$.Let $\mu_i = 1-\chi_i$. Then $1-\chi = \wedge_{i=1}^{\infty}(\chi_i)$. Since χ_i' sare fuzzy supra nowhere dense sets in (X,T*),by proposition 3.6, we have $1-\chi_i$'s are fuzzy supra dense sets in (X,T*).Hence $cl^*(\mu_i)=cl^*(1-\chi_i)=1$,therefore we have $1-\chi = \wedge_{i=1}^{\infty}(\chi_i)$where $cl^*(\mu_i) = 1$.

4 . FUZZY SUPRA BAIRE SPACE

Definition 4.1

Let (X,T*) be a Fuzzy supra topological Spaces. Then (X,T*) is called a Fuzzy supra Baire Spaces if $int^*(\vee_{i=1}^{\infty}(\chi_i)) = 0$, where (χ_i)'s are Fuzzy supra nowhere dense set in (X,T*).

EXAMPLE 4.1: Let X ={a, b, c}. The fuzzy sets λ, μ and γ are defined on X as follows:

λ: X $\rightarrow$[0, 1] is defined as $\lambda(a) = 1$; $\lambda(b)=0.2$; $\lambda(c)=0.7$

μ: X $\rightarrow$ [0, 1] is defined as $\mu(a)=0.3$; $\mu(b) = 1$; $\mu(c) =0.2$

γ: X $\rightarrow$[0, 1] is defined as $\gamma(a) =0.7$; $\gamma(b) =0.4$; $\gamma(c) =1$

Then, $T^* = \{0, \lambda, \gamma, \gamma\vee\lambda, \gamma\wedge\lambda, 1\}$ and $S^* = \{0, \mu, \gamma, \gamma\vee\mu, \gamma\wedge\mu, 1\}$ are fuzzy supra topologies on X and the fuzzy supra nowhere dense sets are $1-\lambda$,$1-\gamma$, $1-(\gamma\vee\lambda)$, $1-(\gamma\wedge\lambda)$) in (X,T*). Now $[(1-\lambda)\vee(1-\gamma)\vee(1-(\gamma\vee\lambda))\vee(1-(\gamma\wedge\lambda))]=1-(\gamma\wedge\lambda)$ and $int^*[1-(\gamma\wedge\lambda)]=0$. Hence (X,T*) is a fuzzy supra Baire space.

On the other hand the fuzzy supra nowhere dense sets are $1-\mu$,$1-\gamma$,$1-(\gamma\vee\mu)$ in (X,S*). Now $[(1-\mu)\vee(1-\gamma)\vee(1-(\gamma\vee\mu))]= 1-(\gamma\wedge\mu)$ and $int^*[1-(\gamma\wedge\mu)]\neq 0$. Hence (X,S*) is not a fuzzy supra Baire space.

Proposition 4.1

Let (X,T*) be a fuzzy supra topological space. Then the following are equivalent:

(i) (X,T*) is a fuzzy supra Baire space.

(ii) $int^*(\chi)=0$, for every fuzzy supra first category Set in χ in (X,T*).

(iii) $cl^*(\mu)= 1$, for every fuzzy supra residual set μin (X,T*).

Proof

(**1**)$\Longrightarrow$(**2**).

Let χbe a fuzzy supra first category set in (X,T*).Then $\chi=\vee_{i=1}^{\infty}(\chi_i)$,where χ_i, s are fuzzy supra nowhere dense sets in (X,T*).Then, we have $int^*(\chi) = int^*(\vee_{i=1}^{\infty}(\chi_i))$. Since(X,T*)is a fuzzy supra Baire space, $int^*(\vee_{i=1}^{\infty}(\chi_i)=0$. Hence $int^*(\chi)=0$ for any fuzzy supra first category set χ in(X,T*).

(**2**)$\Longrightarrow$(**3**).

Let μ be a fuzzy supra residual set in (X,T*). Then $1-\mu$ is a fuzzy supra first category set in (X,T*). By hypothesis, $int^*(1-\mu) =0$. Then $1- cl^*(\mu)=0$. Hence $cl^*(\mu)= 1$ for any fuzzy supra residual set μ in (X,T*).

(**3**)$\Longrightarrow$(**1**).

Let χ be a fuzzy supra first category set in(X,T*). $\chi=\vee_{i=1}^{\infty}(\chi_i)$,where χ_i' s are fuzzy supra nowhere dense sets in (X,T*). Now χ is a fuzzy supra first category set in (X,T*) implies that $(1-\chi)$ is a fuzzy supra residual set in (X,T*). By hypothesis, we have $cl^*(1-\chi)=1$. Then $1- int^*(\chi)=1$.Hence $int^*(\chi)=0$. That is, $int^*(\vee_{i=1}^{\infty}(\chi_i)) =0$ where χ_i' s are fuzzy supra nowhere dense sets in(X,T*). Hence(X,T*) is a fuzzy supra Baire space.

Proposition 4.2

If χ is a fuzzy supra dense set and fuzzy supra G_δ -set in a fuzzy supra topological space (X,T*), then $1-\chi$ is a fuzzy supra first category set in (X,T*).

Proof

Since χ is a fuzzy supra G_δ set in (X,T*), $\chi = \wedge_{i=1}^{\infty}(\chi_i)$ where $\chi_i \in T^*$ and since χ is a fuzzy supra dense in (X,T*), $cl^*(\chi)=1$,then $cl^*(\wedge_{i=1}^{\infty}(\chi_i))=1$.But $cl^*(\wedge_{i=1}^{\infty}(\chi_i)) \leq \wedge_{i=1}^{\infty} cl^*(\chi_i)$.Hence, $1 \leq \wedge_{i=1}^{\infty} cl^*(\chi_i)$.(ie) $\wedge_{i=1}^{\infty} cl^*(\chi_i) =1$. Then we have $cl^*(\chi_i)=1$ for each $\chi_i \in T^*$ and hence $cl^*(int\ \chi_i)=1$.which Implies that $1-cl^*int^*(\chi_i)=0$ and hence $int^*cl^*(1-\chi_i)=0$. Therefore $1-\chi_i$.is a fuzzy supra nowhere dense set in (X,T*). Now $1-\chi_i = 1-\wedge_{i=1}^{\infty}(\chi_i) = \vee_{i=1}^{\infty}(1-\chi_i)$ therefore $1-\chi_i = \vee_{i=1}^{\infty}(1-\chi_i)$ where $(1-\chi_i)$'s are fuzzy supra nowhere dense sets in (X,T*).Hence $1-\chi_i$ is a fuzzy supra first category set in (X,T*).

Proposition 4.3

If the fuzzy supra topological space (X,T*)has a fuzzy supra dense and fuzzy supra G_δ set, then (X,T*) is a fuzzy supra Baire space.

Proof

Let χ be a fuzzy supra dense and G_δ set in (X,T*). Then by proposition3.2, $1-\chi$ is a fuzzy supra first category set in (X,T*) and $(1-\chi) = \vee_{i=1}^{\infty}(\chi_i)$where $(1-\chi)$'s are fuzzy supra nowhere dense sets in (X,T*) But $int^*(1-\chi)=1- cl^*(\chi)=1-1=0$ (since χ is fuzzy supra dense, $cl^*(\chi)=1$). Then $int^*(\vee_{i=1}^{\infty}(1-\chi_i))= int^*(1-\chi) = 0$ and hence (X,T*) is a fuzzy supra Baire space.

Proposition 4.4

If the fuzzy supra topological space (X,T*) is a fuzzy supra first category space, then (X,T*) is not a fuzzy supra baire space.

Proof:

Since (X,T*) is a fuzzy supra first category space, then $v_{i=1}^{\infty}(\chi_i) = 1$, where χ_i's are fuzzy supra nowhere dense sets in (X,T*). Therefore $int^*(v_{i=1}^{\infty}(\chi_i)) = int^*(1) = 1 \neq 0$. Hence $int^*(v_{i=1}^{\infty}(\chi_i)) \neq 0$, where χ_i's are fuzzy supra nowhere dense sets in (X,T*) and therefore (X,T*) is not a fuzzy supra Baire space.

Proposition 4.5

A fuzzy supra topological space (X,T*) is a fuzzy supra Baire space iff $v_{i=1}^{\infty}(\chi_i)) = 1$ where χ_i's are fuzzy supra closed set in (X,T*) with $int^*(\chi_i)=0$,that implies $int^*(v_{i=1}^{\infty}(\chi_i))=0$

Proof:

If (X,T*) is a fuzzy supra Baire space then χ_i's are fuzzy supra closed set in (X,T*) and $int^*(\chi_i)=0$ whrerχ_i are fuzzy supra nowhere dense set in (X,T*).Then$v_{i=1}^{\infty}(\chi_i) = 1$ that implies that 1 is a fuzzy supra first category in (X,T*).Since (X,T*) is a fuzzy supra Baire space then by proposition 4.1, $int^*(1)=0$.Conversely $int^*(v_{i=1}^{\infty}(\chi_i))=0$, where χ_i's are fuzzy supra closed set in (X,T*) with $int^*(\chi_i)=0$.

CONCLUSION

In this paper, we introduced and studied a new concept Fuzzy Supra Baire Space in Fuzzy Supra topological space. This new concept and the characteristic properties are discussed in depth.

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On Fuzzy Supra Regular Baire Spaces

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ABSTRACT. In this paper, the concepts of FSR nowhere dense sets are introduced and studied. By means of FSR nowhere dense sets, the concepts of FSR Baire Space(fuzzy supra regular) are introduced and Characterizations of FSR Baire space are studied.

1. INTRODUCTION

In 1937, Stone [6] introduced the notations of regular open sets in topological spaces, In 1981, Azad [2] introduced the concept of FR open sets in FT Spaces. In 1984, Mashhour et al [3] introduced the concept of FST Spaces. In 1987, Abd El-Monsef et al [1] introduced FSRT Space and defined basic notions of FST Space. The concept of Baire Space was first advanced by Baire (1899). The study of Baire Spaces and related concepts became popular among topologists interested in the completeness. In 2013, the concept of Baire Space in fuzzy setting was introduced and studied by G.Thangaraj and S.Anjalmose [7]. In this paper, Several characterizations of FSR Baire Space are obtained.

2. PRELIMINARIES

We give some basic notions and results used in the sequel. In this work by (X, T^*), we will denote a FST Spaces.

Definition 2.1. *[4] A collection δ^* of fuzzy sets in a set X is called FST on X if the following conditions are satisfied:*

(i) *0 and 1 belongs to δ^*.*

(ii) *$g_\chi \in \delta^*$ for each $\chi \in \wedge$ implies $(\vee_{\chi\in\wedge} g_\chi) \in \delta^*$.*

The pair (X, δ^*) is called a FST Space. The elements of δ^* are called FS open sets and the complement of a FS open set is called FS closed set.

Lemma 2.1. *[5] For any fuzzy set χ in a FST space X,*

(i) $1 - cl^*(\chi) = int^*(1 - \chi)$,

(ii) $1 - int^*(\chi) = cl^*(1 - \chi)$.

Theorem 2.1. *[4] For any two fuzzy sets g and h in a FST Space (X, δ^*),*

(1) *g is a FS closed set if and only if $cl^*(g) = g$.*

(2) *g is a FS open set if and only if $int^*(g) = (g)$.*

(3) *$g \leq h$ implies $int^*(g) \leq int^*(h)$ and $cl^*(g) \leq cl^*(h)$.*

(4) *$cl^*(cl^*(g)) = cl^*(g)$ and $int^*(int^*(g)) = int^*(g)$.*

(5) *$cl^*(g \vee h) \geq cl^*(g) \vee cl^*(h)$.*

(6) *$cl^*(g \wedge h) \leq cl^*(g) \wedge cl^*(h)$.*

(7) *$int^*(g \vee h) \geq int^*(g) \vee int^*(h)$.*

(8) *$int^*(g \wedge h) \leq int^*(g) \wedge int^*(h)$.*

(9) *$cl^*(g^c) = [int^*(g)]^c$.*

(10) *$int^*(g^c) = [cl^*(g)]^c$.*

Definition 2.2. *[5] Let (X, T^*) be a FST Space. A fuzzy set g is called a FSR open set if $int^*(cl^*(g)) = g$ and a fuzzy set g is called a FSR closed set if $cl^*(int^*(g)) = g$.*

Definition 2.3. *[5] Let (X, T^*) be a FST Space and g be a fuzzy set in X, then the FSR closure and FSR interior of g is denoted and defined respectively as*

$$rcl^*(g) = \wedge \{ h \mid h \text{ is a FSR closed set in } X \text{ and } g \leq h\}$$
$$rint^*(g) = \vee \{ h \mid h \text{ is a FSR open set in } X \text{ and } h \leq g\}.$$

3. Fuzzy Supra Regular Nowhere Dense Sets

Definition 3.1. *Let (X, T^*) be a FST Space. A fuzzy set χ in (X, T^*) is called a FSR nowhere dense set if there exists no non-zero FSR open set μ in (X, T^*) such that $\mu < cl^*(\chi)$. That is, $int^*cl^*(\chi) = 0$, in (X, T^*).*

Example 1. *Let* $X = \{a, b, c\}$. *The fuzzy sets* χ, μ, γ *and* δ *are defined on* X *as follows:*

$\chi : X \to [0, 1]$ *defined as* $\chi(a) = 0.5;\ \ \chi(b) = 0.7;\ \ \chi(c) = 0.3$

$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.6;\ \ \mu(b) = 0.5;\ \ \mu(c) = 0.8$

$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 0.4;\ \ \gamma(b) = 0.4;\ \ \gamma(c) = 0.7$

$\delta : X \to [0, 1]$ *defined as* $\delta(a) = 0.7;\ \ \delta(b) = 0.3;\ \ \delta(c) = 0.6$ *Then* $T^* = \{0, \chi, \mu, \gamma, \delta, (\chi \vee \mu), (\gamma \vee \delta), (\chi \vee \delta), (\chi \vee \gamma), (\mu \vee \delta), (\chi \wedge \mu), (\gamma \wedge \delta), (\chi \wedge \delta), (\chi \wedge \gamma), (\mu \wedge \delta), 1\}$ *is a FST on* X. *The non-zero FSR open sets in* (X, T^*) *are* $\chi, \mu, \gamma, \delta, (\chi \vee \mu), (\gamma \vee \delta), (\chi \vee \delta), (\chi \vee \gamma), (\mu \vee \delta), (\chi \wedge \mu), (\gamma \wedge \delta), (\chi \wedge \delta), (\mu \wedge \delta)$. *Now the fuzzy sets* $int^*cl^*(\chi) = \chi \neq 0$ *and* $int^*cl^*(\gamma) = int[1 - (\chi \vee \mu)] = 0$ *in* (X, T^*). *Hence* χ *is not a FSR nowhere dense set in* (X, T^*) *and* γ *in a FSR nowhere dense set in* (X, T^*).

Remark 3.1.

(i) *If* α *and* β *are FSR nowhere dense sets in a FST Space* (X, T^*), *then* $\alpha \vee \beta$ *need not be a FSR nowhere dense set in* (X, T^*).

(ii) *The complement of a FSR nowhere dense set in a FST Space need not be a FSR nowhere dense set. For consider the following example.*

Example 2. *Let* $X = \{a, b, c\}$. *The fuzzy sets* χ, μ *and* γ *are defined on* X *as follows:*

$\chi : X \to [0, 1]$ *defined as* $\chi(a) = 0.3;\ \ \chi(b) = 0.1;\ \ \chi(c) = 0.2$

$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.7;\ \ \mu(b) = 0.4;\ \ \mu(c) = 0.1$

$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 0.1;\ \ \gamma(b) = 0.2;\ \ \gamma(c) = 0.7$.

Then $T^* = \{0, \chi, \mu, \gamma, (\mu \vee \gamma), (\mu \wedge \gamma), 1\}$ *is a FST on* X. *On Computation, one can see that FSR nowhere dense sets in* (X, T^*) *are* $1 - \mu$, $1 - \gamma$ *and* $1 - \mu \vee \gamma$. *Now* $(1-\mu) \vee (1-\gamma) = 1 - \mu \wedge \gamma$ *and* $int^*cl^*[(1-\mu) \vee (1-\gamma)] = int^*(1 - \mu \wedge \gamma) = \chi \neq 0$ *and hence* $(1 - \mu) \vee (1 - \gamma)$ *is not a FSR nowhere dense set in* (X, T^*) *and* $1 - \mu$ *is a FSR nowhere dense set in* (X, T^*) *where as* χ *is not a FSR nowhere dense set in* (X, T^*).

Definition 3.2. *A fuzzy set* χ *in a FST Space* (X, T^*) *is called a FSR dense if there exists no FSR closed set* μ *in* (X, T^*) *such that* $\chi < \mu < 1$. *That is,* $cl^*(\chi) = 1$, *in* (X, T^*).

Proposition 3.1. *If* χ *is a FSR nowhere dense set in a FST Space* (X, T^*), *then* $1 - \chi$ *is a FSR dense set in* (X, T^*).

Proof. Let χ be a FSR nowhere dense set in (X, T^*). Then $int^*cl^*(\chi) = 0$, in (X, T^*). Now $1-int^*cl^*(\chi) = 1-0 = 1$ and hence by lemma 2.1, $cl^*int^*(1-\chi) = 1$ in (X, T^*). But $cl^*int^*(1-\chi) \leq cl^*(1-\chi)$ implies that $1 \leq cl^*(1-\chi)$. That is, $cl^*(1-\chi) = 1$ in (X, T^*). Therefore $1-\chi$ is a FSR dense set in (X, T^*). □

Proposition 3.2. *If χ is a FSR closed set with $int^*(\chi) = 0$, in a FST Space (X, T^*), then χ is a FSR nowhere dense set (X, T^*).*

Proof. Let χ be a FSR closed set with $int^*(\chi) = 0$, in (X, T^*). Then $cl^*(\chi) = \chi$ and $int^*(\chi) = 0$ and hence $int^*cl^*(\chi) = int^*(\chi) = 0$ in (X, T^*). Therefore χ is a FSR nowhere dense set (X, T^*). □

Proposition 3.3. *If χ is a non-zero fuzzy set in a FST Space (X, T^*), then $\chi \vee int^*cl^*int^*\chi \leq cl^*\chi$, in (X, T^*).*

Proof. Since $cl^*(\chi)$ is a FSR closed set in (X, T^*), $int^*cl^*int^*(cl^*(\chi)) \leq cl^*(\chi)$. Then $int^*cl^*int^*(\chi) \leq int^*cl^*int^*(cl^*(\chi))$, $\chi \leq cl^*(\chi)$ and hence $\chi \vee int^*cl^*int^*(\chi) \leq cl^*(\chi)$, in (X, T^*). □

Proposition 3.4. *If χ is a FSR nowhere dense set in a FSR topological space (X, T^*), then $int^*(\chi) = 0$ in (X, T^*).*

Proof. Let χ be a FSR nowhere dense set in (X, T^*). Then $int^*cl^*(\chi) = 0$ in (X, T^*). Now $int^*(\chi) \leq int^*cl^*(\chi)$, implies that $int^*(\chi) \leq 0$, in (X, T^*). That is, $int^*(\chi) = 0$ in (X, T^*). □

Proposition 3.5. *If $\chi \leq \mu$ and μ is a FSR nowhere dense set in FST space (X, T^*), than χ is also a FSR nowhere dense set in (X, T^*).*

Proof. Now $\chi \leq \mu$, implies that $cl^*(\chi) \leq int^*cl^*(\mu)$ in (X, T^*). Since μ is a FSR nowhere dense set in (X, T^*). That is $int^*cl^*(\mu) = 0$. Then $int^*cl^*(\chi) \leq 0$ in (X, T^*). That is $int^*cl^*(\chi) = 0$ in (X, T^*). Therefore χ is a FSR nowhere dense set in (X, T^*). □

Proposition 3.6. *If either χ or μ is a FSR nowhere dense set in FST space (X, T^*), than $\chi \wedge \mu$ is a FSR nowhere dense set in (X, T^*).*

Proof. $Int^*cl^*(\chi \wedge \mu) \leq int^*[cl*(\chi) \wedge cl^*(\mu)] = int^*cl^*(\chi) \wedge int^*cl^*(\mu)$ in (X, T^*). If either χ or μ is a FSR nowhere dense set in (X, T^*), then either $int^*cl^*(\chi) = 0$ or $int^*cl^*(\mu) = 0$. Then either $int^*cl^*(\chi \wedge \mu) \leq 0 \wedge int^*cl^*(\mu) = 0$ (or) $int^*cl^*(\chi \wedge \mu) \leq int^*cl^*(\chi) \wedge 0 = 0$. Therefore, $\chi \wedge \mu$ is a FSR nowhere dense set in (X, T^*). □

Proposition 3.7. *If χ is a FSR dense set in FST space (X, T^*), than χ is a FSR open set in (X, T^*).*

Proof. Let χ be a FSR dense set in (X, T^*). Then $cl^*(\chi) = 1$, in (X, T^*). Since $cl^*(\chi) \leq cl^*(\chi), 1 \leq cl^*(\chi)$. That is $cl^*(\chi) = 1$, in (X, T^*). Now, $cl^*int^*cl^*(\chi) = cl^*int^*(\chi) = cl^*(\chi) = 1$ and hence $\chi \leq cl^*int^*cl^*(\chi)$ in (X, T^*). Therefore, χ is a FSR open set in (X, T^*). □

Definition 3.3. *A fuzzy set χ in a FST space (X, T^*) is called a FSR first category set if $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)'s are FSR nowhere dense set in (X, T^*). Any other fuzzy set in (X, T^*) is said to be FSR second category.*

Definition 3.4. *Let χ be a FSR first category set in a FST space (X, T^*). Then $1-\chi$ is called a FSR residual set in (X, T^*).*

4. Fuzzy Supra Regular Baire Spaces

Definition 4.1. *Let (X, T^*) be a FST space. Then (X, T^*) is called a FSR Baire space $int^*(\vee_{i=1}^{\infty}(\chi_i)) = 0$, where (χ_i)'s are FSR nowhere dense set in (X, T^*).*

Example 3. *Let $X = \{a, b, c\}$. The fuzzy sets $\chi, \mu, \gamma, \alpha, \beta, \delta$ and η are defined on X as follows:*

$\chi : X \to [0, 1]$ *defined as* $\chi(a) = 0.5;\ \ \chi(b) = 0.4;\ \ \chi(c) = 0.6$
$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.4;\ \ \mu(b) = 0.6;\ \ \mu(c) = 0.3.$
$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 0.6;\ \ \gamma(b) = 0.5;\ \ \gamma(c) = 0.7$
$\alpha : X \to [0, 1]$ *defined as* $\alpha(a) = 0.3;\ \ \alpha(b) = 0.1;\ \ \alpha(c) = 0.7$
$\beta : X \to [0, 1]$ *defined as* $\beta(a) = 0.6;\ \ \beta(b) = 0.6;\ \ \beta(c) = 0.7$
$\delta : X \to [0, 1]$ *defined as* $\delta(a) = 0.5;\ \ \delta(b) = 0.6;\ \ \delta(c) = 0.6$
$\eta : X \to [0, 1]$ *defined as* $\eta(a) = 0.5;\ \ \eta(b) = 0.5;\ \ \eta(c) = 0.6$

Then $T^* = \{0, \chi, \mu, \gamma, \alpha, \beta, \delta, \eta, 1\}$ is a FST on X. On computations, one can see that the non-zero FSR open set in (X, T^*) are $\chi, \gamma, 1-\mu, \alpha, 1-\alpha, \delta, \eta, 1$. Now the FSR nowhere dense set in (X, T^*) are $1-\delta, 1-\eta, 1-\beta, \mu, 1-\chi$. On computations, $int^*[(1-\delta) \vee (1-\eta) \vee (1-\beta) \vee (\mu) \vee (1-\chi)] = int^*(1-\chi) = 0$. Hence the FST Space (X, T^*) is a FSR Baire Space.

Proposition 4.1. *If $int^*(\vee_{i=1}^{\infty}(\chi_i)) = 0$, where $int^*(\chi_i) = 0$ and (χ_i)'s are FSR closed sets in a FST space (X, T^*), then (X, T^*) is a FSR Baire space.*

Proof. Let (χ_i)'s be FSR closed sets in (X, T^*). Since $int^*(\chi_i) = 0$, by proposition 3.2, the (χ_i)'s are FSR nowhere dense sets in (X, T^*). Therefore $int^*(\vee_{i=1}^{\infty}(\chi_i)) = 0$, where (χ_i)'s are FSR nowhere dense in (X, T^*), implies that (X, T^*) is a FSR Baire space. □

Proposition 4.2. *If $cl^*(\wedge_{i=1}^{\infty}(\chi_i)) = 1$, where (χ_i)'s are FSR dense and FSR open sets in a FST space (X, T^*), then (X, T^*) is a FSR Baire space.*

Proof. Now, $cl^*(\wedge_{i=1}^{\infty}(\chi_i)) = 1$, implies $1 - cl^*(\wedge_{i=1}^{\infty}(\chi_i)) = 0$. Then $int^*[1 - \wedge_{i=1}^{\infty}(\chi_i)] = 0$ in (X, T^*). This implies that $int^*[(\vee_{i=1}^{\infty}(1 - \chi_i)] = 0$. Since (χ_i)'s are FSR dense in (X, T^*), $cl^*(\chi_i) = 1$ and $int^*(1 - \chi_i) = 1 - cl^*(\chi_i) = 1 - 1 = 0$. Hence $int^*(\vee_{i=1}^{\infty}(1-\chi_i)) = 0$, where $int^*(1-\chi_i) = 0$ and $(1-\chi_i)$'s are FSR closed sets in (X, T^*). Then by proposition 4.1, the FST space (X, T^*) is a FSR Baire space. □

Proposition 4.3. *Let (X, T^*) be a FST space. Then the following are equivalent.*

(1) *(X, T^*) is a FSR Baire space.*
(2) *$int^*(\chi) = 0$ for every FSR first category set χ in (X, T^*).*
(3) *$cl^*(\mu) = 1$ for every FSR residual set μ in (X, T^*).*

Proof.

$(\mathbf{1}) \Rightarrow (\mathbf{2})$ Let χ be a FSR first category set in (X, T^*). Then $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)'s are FSR nowhere dense set in (X, T^*). Now $int^*(\chi) = int^*(\vee_{i=1}^{\infty}(\chi_i))$ (Since (X, T^*). is a FSR Baire space). Therefore $int^*(\chi) = 0$, in (X, T^*).

$(\mathbf{2}) \Rightarrow (\mathbf{3})$ Let μ be a FSR residual set in (X, T^*). Then, $1 - \mu$ is a FSR first category set in (X, T^*). By hypothesis, $int^*(1 - \mu) = 0$, in (X, T^*). This implies that $1 - cl^*(\mu) = 0$ and hence $cl^*(\mu) = 1$, in (X, T^*).

$(\mathbf{3}) \Rightarrow (\mathbf{1})$ Let χ be a FSR first category set in (X, T^*). Then $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)'s are FSR nowhere dense set in (X, T^*). Since χ is a FSR first category set in (X, T^*), $1-\chi$ is a FSR residual set in (X, T^*). By hypothesis, $cl^*(1-\chi) = 1$. Then $1-int^*(\chi) = 1$, in (X, T^*). This implies that $int^*(\chi) = 0$, in (X, T^*). Hence $int^*(\vee_{i=1}^{\infty}(\chi_i)) = 0$, where (χ_i)'s are FSR nowhere dense in (X, T^*), implies that (X, T^*) is a FSR Baire space. □

Definition 4.2. *A FST space (X, T^*) is said to be of FSR first category space if $\vee_{i=1}^{\infty}(\chi_i) = 1_X$, where (χ_i)'s are FSR nowhere dense set in (X, T^*). Otherwise (X, T^*) will be called a FSR second category space.*

Proposition 4.4. *If a FST space (X, T^*) is a FSR Baire space, then (X, T^*) is a FSR second category space.*

Proof. Let (X, T^*) is a FSR Baire space. Then,$int^*(\vee_{i=1}^{\infty}(\chi_i)) = 0$, where (χ_i)'s are FSR nowhere dense in (X, T^*). Then $\vee_{i=1}^{\infty}(\chi_i) \neq 1_X$ [Otherwise, $\vee_{i=1}^{\infty}(\chi_i) = 1_X$, implies that $int^*(\vee_{i=1}^{\infty}(\chi_i)) = int^*(1_X) = 1_X \neq 0$,a contradiction]. Hence (X, T^*) is a FSR second category space. □

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On Fuzzy Supra Hyperconnected Spaces

E. Poongothai, A. Celine

Abstract. This paper is the study of Fuzzy supra hyperconnected spaces (denoted by FSHS). After studying the characteristic of FSHS concerned with different concepts. The relationship between these concepts is investigated. The properties of FSHS are studied.

1. Introduction

In 1984, Mashhour et al [5] introduced the concept of Fuzzy supra topological spaces(denoted by FSTS). In 1987, Abd El-Monsef et al [1] introduced FSTS and defined basic notions of FSTS.

In 1978, The concept of hyper connectedness in topological spaces has been introduced by Steen and Seebach [10]. In 1992, Ajmal *et al.*[2] have studied some of the characterizations and basic properties of hyper connected space. In 2002, Caladas *et al.*[3], introduced and studied the concept of fuzzy hyper connected space.

The main seek of this paper is to extend of the idea of hyperconnected to FSTS and study its characterizations. In section 3 we introduce the notion of FSHS, FS D-Space. The properties of FSHS are investigated.

2. Preliminaries

Definition: 2.1 [5]

A collection δ^* of fuzzy sets in a set Y is called FST on Y if the following conditions are satisfied:

1) **0** and **1** belongs to δ^*.

2) $g_\chi \in \delta^*$ for each $\chi \in \wedge$ implies $(\vee_{\chi \in \wedge} g_\chi) \in \delta^*$.

The pair (Y, δ^*) is called a FSTS. The elements of δ^* are called FS open sets and the complement of a FS open set is called FS closed set.

Definition: 2.2 [5]

Let (Y, δ^*) be a FSTS and g be a fuzzy set in Y, then the FS closure and FS interior of g defined respectively as

$cl^*(g) = \wedge\{h/h$ is a FS closed set in Y and $g \leq h\}$

$int^*(g) = \vee\{h / h$ is a FS open set in Y and $h \leq g\}$

Remark: 2.1 [5]

1) The FS closure of a fuzzy set g in a FSTS is the smallest FS closed set containing h.
2) The FS interior of a fuzzy set g in a FSTS is the largest FS open set contained in h.
3) If (Y, δ*) is an associated FSTS with the fuzzy topological space (Y, δ) and h is any fuzzy set in Y, then int (h) ≤ int *(h) ≤ h ≤ cl *(h) ≤ cl (h).

Theorem: 2.1[5]

For any two fuzzy sets g and h in a FSTS (Y,T*),

1) g is a FS closed set if and only if cl *(g) = g.
2) g is a FS open set if and only if int*(g) = (g).
3) g ≤ h implies int*(g) ≤ int *(h) and cl*(g) ≤ cl *(h)
4) cl*(cl*(g)) = cl*(g) and int*(int*(g)) = int*(g).
5) cl*(g v h) ≥ cl*(g) v cl *(h)
6) cl*(g ∧ h) ≤ cl*(g) ∧ cl*(h)
7) int*(g v h) ≥ int*(g) v int *(h)
8) int*(g ∧ h) ≤ int*(g) ∧ int *(h)
9) cl*(g ^{c}) = [int*(g)]^c
10) int*(g ^{c}) = [cl*(g)]^c

Definition 2.3[3]

A FTS (Y ,T) is said to be FHC if every non-null fuzzy open subset of (Y, T) is fuzzy dense in (Y,T) .

Definition 2.4 [8]

Let (Y,T*) be a FSTS. A fuzzy set μ in (Y,T*) is called a FS-nowhere dense set if there exists no non-zero FS open set δ in (Y,T*) such that δ<cl*(μ). That is, int*cl*(μ)=0, in (Y,T*).

Definition 2.5 [8]

A fuzzy set μ in a FSTS (Y,T*) is called a FS G_δ -set in (Y,T*) if $\mu = \wedge_{i=1}^{\infty} (\mu_i)$ where $\mu_i \epsilon T$ for $i \epsilon I$.

Definition 2.6 [8]

A fuzzy set μ in a FSTS (Y,T*) is called a FS dense set if there exists no FS closed set δ in (Y,T*) such that μ<δ<1. That is, cl*(μ)=1.

Definition 2.7 [7]

A fuzzy set χ in a FSTS (Y,T*) is called a FS first category set if $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)'s are FSR nowhere dense set in (Y,T*). Any other fuzzy set in (Y,T*) is said to be FS second category.

3. Fuzzy Supra Hyperconnected Spaces

Definition3.1

A FSTS (Y,T*) is said to be fuzzy supra hyper connected if every non-null FS open set of (Y, T*) is FS dense is (Y, T*).

Example3.1

Let Y={a,b,c}. The FS sets δ, ω and γ are defined on Y as follows;

δ:Y→[0,1] defined as δ(a)=0.5; δ(b)=0.7; δ(c)=0.7
ω:Y→[0,1] defined as ω(a)=0.6; ω(b)=0.5; ω(c)=0.7
γ:Y→[0,1] defined as γ(a)=0.4; γ(b)=0.6; γ(c)=0.5

Then T*={0, δ, ω, γ, δ∧ω, γ∧ω, ω∧γ, δ∨ω,ω∨γ,1} are open sets in a FSTS on Y. Here every fuzzy open set is FS dense set. Therefore the FSTS (Y, T*) is FSHS.

Proposition 3.1

In a FSTS (Y, T*), the following conditions are equivalent:

i) (Y,T*) is FSHS.
ii) Every fuzzy set of (Y,T*) is either FS dense or FS nowhere dense set.

Proof:

(i) $\Rightarrow$ (ii)

Let (Y,T*) be a FSHS and δ be any fuzzy set such that $\delta \leq 1_Y$ suppose that δ is not FS nowhere dense, then we have $cl^*(1\text{-}cl^*(\delta)) = 1\text{-}int^*(cl^*(\delta)) \neq 1_Y$. Since $int^*(cl^*(\delta)) \neq 0_Y$ and this implies that $cl^*(int^*(cl^*(\delta)) = 1_Y$. But $cl^*(int^*(cl^*(\delta)) = 1_Y \leq cl^*(\delta)$,which gives that $cl^*(\delta) = 1_Y$. Hence δ is a FS dense set.

(ii) $\Rightarrow$ (i)

Let δ be any non empty FS open set in Y. Now for any non empty FS open set ω, we have $\omega < int^*cl^*(\omega)$ which implies that ω is not FS nowhere dense set but using hypothesis, we have ω is FS dense set. Hence the proof.

But the above equivalent condition is partially true.

Proposition 3.2

In any FSTS (Y,T*), every pair of nonempty FS open set has a nonempty intersection.

Proof:

Suppose that $\delta \wedge \omega = 0$, for any two nonempty FS open sets δ and ω of (Y,T*). This implies that $cl^*(\delta) \wedge \omega = 0_Y$ and δ is not a FS dense set. Since δ is FS open, then $0_Y \neq \delta \leq int^*(cl^*(\delta))$ and δ is not a FS dense set. This is a contradiction. Thus, $\delta \wedge \omega \neq 0_Y$, for every non empty FS open sets δ and ω of (Y,T*).

Remark 3.1

In a FSHS (Y,T*) if δ and ω are two FS open sets then $cl^*(\delta \wedge \omega) = cl^*((\delta) \wedge cl^*(\omega)$.

Example 3.2

Let Y={a,b,c}. The FS sets δ, ω and γ are defined on Y as follows;

δ: Y→[0,1] defined as δ(a)=0.7; δ(b)=0.6; δ(c)=0.6

ω: Y→[0,1] defined as ω(a)=0.6; ω(b)=0.8; ω(c)=0.7

γ: Y→[0,1] defined as γ(a)=0.7; γ(b)=0.5; γ(c)=0.9

Then T*={0, δ, ω, γ, δ∧ω, γ∧ω, ω∧γ, δ∨ω,ω∨γ, γ∨ δ, 1} is a FST on X. The non- zero FS intersection sets in (Y,T*) are δ,ω, γ, δ∧ω, γ∧ω, ω∧γ, δ∨ω,ω∨γ, γ∨ δ. Then the FS intersection sets are dense .Hence FSTS (Y,T*) is FSHS.

Proposition 3.3

Let (Y,T*) be a FSTS. Then (Y,T*) is FSHS, iff 1_Y and 0_Y are the only FSR open sets in Y.

Proof:

Let(Y,T*) be a FSHS. Suppose that ω is a nonempty FSR open set. We have $\omega = int^*cl^*(\omega.)$ This implies that $int^*cl^*(1\text{-}\omega) = cl^*(1\text{-}cl^*(\omega)) = 1\text{-}\omega \neq 1_Y$ since $\omega \neq 0_Y$. But this is a contradiction to the assumption. Thus1_Y and 0_Y are the only FSR open sets in Y. Suppose that Y is not FSHS. This implies that there exist a nonempty FS open set ω of Y such that $cl^*(\omega) \neq 1_Y$. We have $cl^*(int^*(\omega)) \neq 1_Y$. This implies that the only possibility is, $cl^*(int^*(\omega)) = 0_Y$. Therefore we have $cl^*(\omega) = 0_Y$ where $\omega \neq 0_Y$ which is absurd and hence the FSTS (Y,T*) is a FSHS.

Proposition 3.4

In a FSHS (Y,T*), any fuzzy set δ of Y is FS semi open set if $int^*(\delta) \neq 0$.

Proof:

Let Y be a FSHS and δ be any fuzzy set of Y where $int^*(\delta) \neq 0$. Therefore $cl^*(int^*(\delta) = 1$, Thus $\delta \leq cl^*(int^*(\delta))$. Hence, every fuzzy set δ is a FS semi open.

Proposition 3.5

In a FSHS (Y,T*), finite intersection of FS semi-open sets is FS semi-open set.

Proof:

Let us consider that δ and ω are two nonempty FS semi-open sets in a FSHS (Y,T*). So $\delta \leq cl^*(int^*(\delta)$ and $\omega \leq cl^*(int^*(\omega)$. Therefore $cl^*(\delta) = cl^*(int^*(\delta))=1$ and $cl^*(\omega)= cl^*(int^*(\omega)=1$. Since δ and ω are two nonempty FS semi-open sets in a FSHS and so $\delta \wedge \omega \neq 0$.

Proposition 3.6

In a FSHS (Y,T*), finite intersection of FS open sets is FS open set.

Proof:

Let us consider that δ and ω are two nonempty FS open sets in a FSHS (Y,T*). So, $\delta \leq cl^*(int^*(\delta))$ and $\omega \leq cl^*(int^*(\omega))$. Therefore $cl^*(\delta)= cl^*(int^*(\delta))=1$ and $cl^*(\omega)= cl^*(int^*(\omega))=1$. Since δ and ω are two nonempty FS open sets in a FSHS and so $\delta \wedge \omega \neq 0$. This implies that $cl^*(int^*(\delta \wedge \omega))= cl^*(int^*(\delta)) \wedge cl^*(int^*(\omega))=1$. Therefore $\delta \wedge \omega \leq cl^*(int^*(\delta)) \wedge cl^*(int^*(\omega))= cl^*(int^*(\delta \wedge \omega))$. This implies that $\delta \wedge \omega$ is a FS open set.

Definition 3.2

A FSTS (Y,T*) is called a FS P-Space if countable intersection of FS open sets in (Y,T*) is fuzzy open. That is, every non-zero FS G_δ-set in (Y,T*), is FS open in (Y,T*).

Definition 3.3

A FSTS (Y,T*) is called a FS first category if $\vee_{i=1}^{\infty}(\delta_i)=1$, where (δ_i)'s are FS nowhere dense set in (Y,T*). A topological space which is not of FS first category, is said to be FS second category.

Proposition 3.7

If a FS P-space (Y,T*) is a FSHS, then (Y,T*) is a FS second category.

Proof:

Let the FS P-space (Y,T*) be a FSHS, then (Y,T*) is a FS Baire space and hence $int^*(\vee_{i=1}^{\infty}(\delta_i))=0$,where (δ_i)'s are FS nowhere dense sets in (Y,T*), we claim that $\vee_{i=1}^{\infty}(\delta_i) \neq 1$, suppose that $\vee_{i=1}^{\infty}(\delta_i)=1$.This implies $int^*(\vee_{i=1}^{\infty}(\delta_i))=int(1)=0 \neq 0$,a contradiction. Hence $\vee_{i=1}^{\infty}(\delta_i) \neq 1$,where (δ_i)'s are FS nowhere dense sets in (Y,T*). Therefore (Y,T*) is a FS second category.

Proposition 3.8

Let (Y,T*) be a FSTS. The following conditions are equivalent

i) (Y,T*) is FSHS.

ii) Every FS pre open set is FS dense set.

Proof:

(i) $\Rightarrow$ (ii)

Let us suppose that δ is any FS pre open set. This implies that $\delta \leq int^*cl^*(\delta)$. We have $cl^*(\delta)= cl^*(int^*(cl^*(\delta))=1$. Therefore δ is FS dense set.

(ii) $\Rightarrow$ (i)

Let δ be any FS pre open set. So $\delta \leq int^*cl^*(\delta)$, δ is FS dense set. Therefore $cl^*(\delta)= cl^*(int^*(cl^*(\delta))=1$. This implies that (Y,T*), is FSHS.

Definition 3.4

A FSTS (Y,T*) is said to be FS D-Space, if every non-null FS open set in X is a FS dense.

Proposition 3.9

Every FS D-space is a FSHS.

Proof:

Let (Y,T*) be a FS D-space. Suppose that δ and ω are non-null FS open sets in (Y,T*) such that $\delta \wedge \omega=0$. This implies that $cl^*(\delta) \wedge \omega=0$ and δ is not a FS dense set. Since δ is FS open, then $0 \neq \delta \leq int^*(cl^*(\delta))$ and δ is not a FS dense set. This is a contradiction. Thus $\delta \wedge \omega \neq 0$. So (Y,T*) is FSHS.

Proposition 3.10

If (Y,T*) is FS D-space, then every FS set δ in Y is FS dense or FS nowhere dense.

Proof:

Let (Y,T*) be a FS D-space and δ be a FS set of Y. Suppose that δ is not FS nowhere dense, then $int^*(cl^*(\delta))\neq 0$. Since $int^*(cl^*(\delta))$ is non-null FS open set of Y, then $cl^*(int^*(cl^*(\delta)))=1$. Since $1= cl^*(int^*(cl^*(\delta)))\leq cl^*(cl^*(\delta))= cl^*(\delta)$, $cl^*(\delta)=1$. Thus δ is FS dense in (Y,T*).

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Fuzzy Supra Strongly Hyper Connected Spaces and Fuzzy Supra Baire Spaces

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Abstract: In this paper, the concept of fuzzy supra strongly hyperconnected spaces are studied. Finding the relations between fuzzy supra strongly hyperconnected spaces and other fuzzy supra topological spaces. Illustrate the concept with suitable examples.

Keywords **- Fuzzy supra nowhere dense set, Fuzzy supra first category, Fuzzy supra P-space, Fuzzy supra submaximal space, Fuzzy supra hyperconnected spaces, Fuzzy supra Baire spaces.**

I. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by L.A.Zadeh [11] in 1965. By applying the fuzzy set notions to general topology, C.L.Chang [4] introduced the theory of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concept of hyperconnectedness in topological spaces has been introduced by Steen and Seebach [10] in 1970. In 2002, Caldas et.al.[3] defined fuzzy hyperconnectedness in fuzzy topological space. In 2017, T.M.Al-shami [2] introduced the concept of strongly hyperconnected spaces in classical topology. The concept of fuzzy supra hyperconnected spaces was introduced and studied by this author in [6]. In this paper, introduce the concept of fuzzy supra strongly hyperconnected spaces. Finding the relations between fuzzy supra strongly hyperconnected spaces and other fuzzy supra topological spaces. Illustrate the concept with suitable examples.

II. PRELIMINARIES

Definition 2.1 [8]

A fuzzy set λ in a fuzzy supra topological space (X,T^*) is called a fuzzy supra dense if there exists no fuzzy supra closed set μ in (X,T^*) such that $\lambda < \mu < 1$. That is, $cl^*(\lambda)=1$, in (X,T^*).

Definition 2.2 [8]

A fuzzy set λ in fuzzy supra topological space (X,T^*) is called a fuzzy supra nowhere dense set if there exists no non-zero fuzzy supra open set μ in (X,T^*) such that $\mu < cl^*(\lambda)$. That is, $int^*cl^*(\lambda)=0$, in (X,T^*).

Lemma 2.1 [1]

For a fuzzy set λ of a fuzzy supra topological space (X,T^*),

(i) $1 - int^*(\lambda) = cl^*(1-\lambda)$,

(ii) $1 - cl^*(\lambda) = int^*(1-\lambda)$.

Definition 2.3 [6]

A fuzzy set λ in a fuzzy supra topological space (X,T^*) is called a fuzzy supra first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy supra nowhere dense set in (X,T^*). Any other fuzzy set in (X,T^*) is said to be fuzzy supra second category.

Definition 2.4 [6]

A fuzzy supra topological space (X,T^*) is called a fuzzy supra P- space if every non zero fuzzy supra G_δ - set in (X,T^*) is fuzzy supra open in (X,T^*). That is, if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in (X,T^*)$ for $i \in \wedge$.

Definition 2.5 [7]

A fuzzy set λ in a fuzzy supra topological space (X,T^*) is called a fuzzy supra G_δ - set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T^*$ for $i \in I$

Definition 2.6 [7]

A fuzzy supra topological space (X,T^*) is called a fuzzy supra Baire spaces if $int^*(\vee_{i=1}^{\infty}(\lambda_i))=0$, where (λ_i)'s are fuzzy supra nowhere dense set in (X,T^*).

III. FUZZY SUPRA HYPERCONNECTED SPACE

Definition 3.1 [6]

A fuzzy supra topological space (X, T^*) is said to be fuzzy supra hyper connected if every non- null fuzzy supra open set of X is fuzzy supra dense in (X, T^*).

Definition 3.2 [5]

A fuzzy set λ in fuzzy supra topological space (X,T^*) is called a fuzzy supra semiopen if there exists a fuzzy supra open set $\mu \epsilon T^*$ such that $\mu \leq \lambda \leq cl^*(\mu)$.

Definition 3.3

In a fuzzy supra topological space (X,T^*) is said to be fuzzy supra connected if X cannot be represented as the union of two non empty disjoint open fuzzy sets.

Definition 3.4

A fuzzy supra topological space (X,T^*) is called a fuzzy supra extremally disconnected space if $cl^*(\lambda)= \lambda$, where $\lambda \epsilon T^*$.

Example 3.1:

Let X = {a, b, c}. Then the fuzzy sets α, β and γ are defined on X as follows:

$\alpha : X \rightarrow [0,1]$ defined as $\alpha(a) = 0.4$, $\alpha(b) = 0.6$, $\alpha(c) = 0.7$.

$\beta: X \rightarrow [0,1]$ defined as $\beta(a) = 0.3$, $\beta(b) = 0.2$, $\beta(c) = 0.7$.

$\gamma: X \rightarrow [0,1]$ defined as $\gamma(a) = 0.6$, $\gamma(b) = 0.4$, $\gamma(c) = 0.3$.

Then, $T^* = \{0, \alpha, \gamma, \alpha \vee \beta, 1\}$ is a fuzzy supra open sets in (X,T^*). Now, we see that $cl^*(\alpha)= \alpha$, $cl^*(\gamma)= \gamma$ and $cl^*(\alpha \vee \beta)= \alpha \vee \beta$. Therefore (X,T^*) is fuzzy supra extremally disconnected space.

Proposition 3.1

Every fuzzy supra hyper connected space (X,T^*) is fuzzy supra extremally disconnected.

Proof.

Suppose that (X,T^*) is fuzzy supra hyper connected space. Then for any fuzzy supra open set β, $cl^*(\beta)=1_x$, which implies that $cl^*(\beta)$ is fuzzy supra dense and as a consequence the space (X,T^*) is fuzzy supra extremally disconnected**.**

Remark 3.1

The converse of the above proposition need not be true. That is, Every fuzzy supra extremally disconnected space (X,T^*) is need not be a fuzzy supra hyper connected space.

For consider the example 3.1, (X,T^*) is fuzzy supra extremally disconnected space. But $cl^*(\alpha)= cl^*(\gamma)= cl^*(\alpha \vee \beta) \neq 1$. Therefore, $\alpha, \gamma, (\alpha \vee \beta)$ are fuzzy supra open set but not fuzzy supra dense set in (X,T^*). Hence, (X,T^*) is not a fuzzy supra hyperconnected space.

Proposition 3.2

Let (X,T^*) be a fuzzy supra topological space. Then the following conditions are equivalent:

(i) (X,T^*) is fuzzy supra hyperconnected spaces,

(ii) Every fuzzy supra preopen set is fuzzy supra dense set.

Proof.

(i) $\Rightarrow$ (ii)

Let us assume that α is fuzzy supra pre-open set in (X,T^*), this implies that $\alpha \leq int^* cl^*(\alpha)$. By hypothesis, (X,T^*) is fuzzy supra hyper connected space, we have $cl^*(\alpha)= cl^*(int^* cl^* (\alpha))=1_x$. Therefore α is fuzzy supra dense set.

(ii) $\Rightarrow$ (i)

Let α be any fuzzy supra pre-open set, thus $\alpha \leq int^*cl^*(\alpha)$. By hypothesis, given that α is fuzzy supra dense set. Therefore $cl^*(\alpha)= cl^*(int^*cl^*(\alpha))=1_x$. It follows that (X,T^*) is fuzzy supra hyper connected space.

Proposition 3.3

Arbitrary union of fuzzy supra hyper connected set of X is fuzzy supra hyper connected set.

Proof.

Let $\{\alpha_i / i \epsilon \wedge\}$ be a collection of fuzzy supra hyper connected sets of X.

Then, for each $i \epsilon \wedge$, we have α_i,

$\alpha_i \leq cl^*int^*(\alpha_i) \vee int^*cl^*(\alpha_i)$

Then, $\vee\alpha_i \leq \vee (cl^* int^*(\alpha_i) \vee int^*cl^*(\alpha_i))$

$= cl^*int^*(\vee\alpha_i) \vee int^*cl^*(\vee\alpha_i)$ [Since, $\vee cl^*(\alpha_i) \leq cl^*(\vee\alpha_i)$ and $int^*(\alpha_i) \leq int^*(\vee\alpha_i)$]

Now, $cl^*(\vee\alpha_i) \leq cl^*[(cl^*int^*(\vee\alpha_i) \vee (int^*cl^*(\vee\alpha_i)]$

$= [cl^*cl^*int^*(\vee\alpha_i) \vee (cl^*int^*cl^*(\vee\alpha_i)]$ [Inequality holds in respect of closure property]

$= cl^*int^*(\vee\alpha_i) \vee (cl^*int^*cl^*(\vee\alpha_i))$

Therefore, $cl^*int^*(\vee\alpha_i) \vee cl^*int^*cl^*(\vee\alpha_i) = 1$.

Hence, $cl^*(\vee\alpha_i) = 1$.

Implies that $(\vee\alpha_i)$ is a fuzzy supra hyperconnected in a fuzzy supra topological spaces (X,T^*).

Proposition 3.4

A fuzzy supra topological spaces (X, T^*) is fuzzy supra hyper connected space, for every non null fuzzy supra open set α of X, if $cl^*int^*(\alpha) \vee cl^*int^*cl^* (\alpha) \leq 1$.

Proof.

Let α be a fuzzy set in (X,T^*). For every non null fuzzy supra open subset $\alpha \leq cl^*int^*(\alpha) \vee int^*cl^*(\alpha)$. Since α is hyper connected in (X,T^*). Therefore, $cl^*(\alpha)=1$. $cl^*(\alpha) \leq cl^*(cl^*int^*(\alpha) \vee int^*cl^*(\alpha))$, $1=cl^*(cl^*int^*(\alpha) \vee int^*cl^*(\alpha))$. This implies $cl^*(cl^*int^*(\alpha) \vee int^*cl^*(\alpha))=1$ [α is finite fuzzy supra open set, $cl^*(\vee\alpha) \geq \vee cl^*\alpha$]. Hence, $cl^*int^*(\alpha) \vee cl^*int^* cl^*(\alpha) \leq 1$.

Proposition 3.5

A fuzzy supra topological spaces (X, T^*) is fuzzy supra hyper connected for non null finite fuzzy supra closed sets β of X if $cl^*int^* (\beta) \wedge cl^*int^*cl^* (\beta) \leq 1$.

Proof.

Let β is fuzzy supra closed set. Then by definition, we have $\beta \geq cl^*int^*(\beta) \wedge int^*cl^*(\beta)$, $1-\beta \leq 1- [cl^*int^*(\beta) \wedge int^*cl^*(\beta)]$. Therefore, $int^*(1-\beta) \leq int^*[1- (cl^*int^*(\beta) \wedge int^*cl^*(\beta)]$, $1-cl^*(\beta) \leq 1-cl^*[cl^*int^*(\beta) \wedge int^*cl^*(\beta)]$. Since β is fuzzy supra hyper connected,

$cl^*(\beta) = 1$ and $cl^*(\wedge\beta_i) \subseteq \wedge cl^*(\beta_i)$. Therefore, $1-1 \leq 1- cl^*cl^*int^*(\beta) \wedge cl^*int^*cl^*(\beta)$, $0 \leq 1- cl^*int^*(\beta) \wedge cl^*int^*cl^*(\beta)$. This implies, $cl^*int^*(\beta) \wedge cl^*int^*cl^*(\beta) \leq 1$.

IV. FUZZY SUPRA STRONGLY HYPERCONNECTED SPACE

Definition :4.1

In a fuzzy supra topological spaces (X,T^*), a fuzzy set λ is said to be fuzzy supra strongly nowhere dense set, if $\lambda \wedge (1-\lambda)$ is a fuzzy supra nowhere dense set in (X,T^*). That is $int^*\{cl^*[\lambda \wedge (1-\lambda)]\} = 0$, in (X,T^*).

Definition :4.2

In a fuzzy supra topological spaces (X,T^*) a fuzzy set λ is said to be a fuzzy supra strongly first category set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where$(\lambda_i)$'s are fuzzy supra strongly nowhere dense sets in (X,T^*). Any other fuzzy set in (X,T^*) is said to be a fuzzy supra strongly second category set in (X,T^*).

Definition :4.3

A fuzzy supra topological space (X, T^*) is called a fuzzy supra strongly Baire space if $cl^*(\wedge_{i=1}^{\infty}(\lambda_i))=1$, where (λ_i)'s are fuzzy strongly nowhere dense sets in (X,T^*).

Definition 4.4

A fuzzy supra topological space (X, T^*) is called a fuzzy supra strongly hyper connected space, if the following conditions hold:

(i) if λ is a fuzzy supra dense set in (X, T^*), then λ is a fuzzy supra open set in (X,T^*) and

(ii) if λ is a fuzzy supra open set in (X,T^*), then λ is a fuzzy supra dense set in (X,T^*).

Example 4.1:

Let X = {a, b, c}. Then the fuzzy sets α, β and γ are defined on X as follows:

$\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.7$, $\alpha(b) = 0.8$, $\alpha(c) = 0.9$.

$\beta: X \to [0,1]$ defined as $\beta(a) = 0.6$, $\beta(b) = 0.9$, $\beta(c) = 0.8$.

$\gamma: X \to [0,1]$ defined as $\gamma(a) = 0.6$, $\gamma(b) = 0.7$, $\gamma(c) = 0.8$.

Then, $T^* = \{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\wedge\beta, 1\}$, is a fuzzy supra open sets in (X,T^*). All the fuzzy supra open sets are fuzzy supra dense sets in (X,T^*) and all the fuzzy supra dense sets are fuzzy supra open sets in (X,T^*). Hence (X,T^*) is a fuzzy supra strongly hyperconnected space.

Definition 4.5

A fuzzy supra topological space (X,T^*) is called a fuzzy supra submaximal space if for each fuzzy set in (X,T^*) such that $cl^*(\lambda) = 1$, then $\lambda \epsilon T^*$ in (X, T^*).

Proposition 4.1:

If (X, T^*) is a fuzzy supra strongly hyper connected space, then

(i) (X, T^*) is a fuzzy supra hyper connected space,

(ii) (X, T^*) is a fuzzy supra submaximal space.

Proof.

(i) Let μ be a fuzzy supra open set in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyper connected space, the fuzzy supra open set μ is a fuzzy supra dense set in (X, T^*) and hence (X, T^*) is a fuzzy supra hyper connected space.

(ii) Let λ be a fuzzy supra dense set in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyper connected space, the fuzzy supra dense set λ is a fuzzy supra open set in (X,T^*) and hence (X,T^*) is a fuzzy supra submaximal space.

Proposition 4.2

If λ is a fuzzy supra dense set in a fuzzy supra strongly hyper connected space, then $1 - \lambda$ is a fuzzy supra nowhere dense set in (X,T^*)

Proof.

Let λ be a fuzzy supra dense set in (X,T^*). Then, $cl^*(\lambda) = 1$ in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyper connected space, λ is a fuzzy supra open set in (X,T^*) and hence $1-\lambda$ is a fuzzy supra closed set in (X,T^*). Then, $cl^*(1-\lambda)=1-\lambda$, in (X, T^*). This implies that $int^*cl^*(1 - \lambda) = int^*(1-\lambda) = 1- cl^*(\lambda) = 1 - 1 = 0$, in (X,T^*). Hence $1-\lambda$ is a fuzzy supra nowhere dense set in (X,T^*).

Proposition 4.3

If $\lambda=\wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy supra dense sets in a fuzzy supra strongly hyper connected space (X, T^*), then $1 - \lambda$ is a fuzzy supra first category set in (X,T^*).

Proof.

Let $\lambda=\wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s (i = 1 to ∞) are fuzzy supra dense sets in (X,T^*). Then, by proposition 4.2, $(1-\lambda_i)$'s are fuzzy supra nowhere dense sets in (X,T^*). Now $\vee_{i=1}^{\infty}(1-\lambda_i)$ is a fuzzy supra first category set in (X,T^*). But $\vee_{i=1}^{\infty}(1 - \lambda_i) = 1 - \wedge_{i=1}^{\infty}(\lambda_i) = 1-\lambda$. Thus $1-\lambda$ is a fuzzy supra first category set in (X, T^*).

Proposition 4.4

If $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy supra dense sets in a fuzzy supra strongly hyper connected space (X,T^*), then λ is a fuzzy supra residual set in (X,T^*).

Proof.

Let $\lambda=\wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s fuzzy supra dense sets in (X,T^*). Since (X, T^*) is a fuzzy supra strongly hyper connected space, by proposition 4.3, $1 - \lambda$ is a fuzzy supra first category set and hence λ is a fuzzy supra residual set in (X,T^*).

Proposition 4.5

If λ and μ are any two non-zero fuzzy open sets in a fuzzy supra strongly hyper connected space (X, T^*), then $\lambda \wedge \mu \neq 0$, in (X, T^*)

Proof.

Suppose that $\lambda \wedge \mu = 0$, where $\lambda, \mu \epsilon T^*$. Then, $\lambda \leq 1 - \mu$ in (X, T^*). This implies that $cl^*(\lambda) \leq cl^*(1-\mu)$, in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyper connected space, the fuzzy supra open set λ is a fuzzy supra dense set in (X,T^*). Then $1 \leq cl^*(1-\mu)$ in (X,T^*). That is., $cl^*(1-\mu)=1$ and then by lemma 2.1, $1- int^*(\mu)=1$ in (X, T^*). This implies that $int^*(\mu)=0$, a contradiction to $\mu \epsilon T^*$ for which $int^*(\mu)=\mu \neq 0$. Hence, $\lambda \wedge \mu \neq 0$, in (X,T^*).

Proposition 4.6

If $int^*(\lambda)=0$ and $int^*(\mu)=0$ for any two non-zero fuzzy sets defined on X in a fuzzy supra strongly hyperconnected space (X, T^*) then $int^*(\lambda \vee \mu) = 0$, in (X,T^*).

Proof.

Let λ and μ be two non-zero fuzzy sets defined on X such that $int^*(\lambda)=0$ and $int^*(\mu)=0$ in (X,T^*). We want to prove that $int^*(\lambda\vee\mu)=0$. Assume the contrary and suppose that $int^*(\lambda\vee\mu)\neq 0$ in (X,T^*). Then, there exists a fuzzy supra open set δ in(X,T^*) such that $\delta \leq \lambda \vee \mu$. Then $cl^*(\delta) \leq cl^*(\lambda\vee\mu)$ in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyper connected space, for the fuzzy supra open set δ in (X,T^*), $cl^*(\delta) = 1$, in (X,T^*). This implies that $1 \leq cl^*(\lambda \vee \mu)$ in (X,T^*). That is, $cl^*(\lambda \vee \mu) = 1$, in (X,T^*). If $cl^*(\lambda) = 1$, in the fuzzy strongly hyper connected space (X,T^*) then λ will be a fuzzy supra open set in (X,T^*) for which $int^*(\lambda) = \lambda$ and this will contradict the fact that $int^*(\lambda) = 0$. Hence $cl^*(\lambda) \neq 1$,in (X,T^*). Similarly, $cl^*(\mu)\neq 1$, in (X,T^*). Now $1- cl^*(\lambda) \neq 0$ and $1-cl^*(\mu) \neq 0$ in (X,T^*) and $[1-cl^*(\lambda)]\wedge[1-cl^*(\mu)] = 1 - [cl^*(\lambda) \vee cl^*(\mu)] = 1 - cl^*(\lambda\vee\mu) =1 - 1 = 0$. Thus, $1- cl^*(\lambda)$ and $1 - cl^*(\mu)$ are non-zero fuzzy supra open sets in (X,T^*) with $[1- cl^*(\lambda)] \wedge [1- cl^*(\mu)] = 0$.But this is a contradiction [by proposition 4.5]. Hence $int^*(\lambda\vee\mu) = 0$, in (X,T^*).

V. FUZZY SUPRA HYPERCONNECTED SPACE AND FUZZY SUPRA BAIRE SPACE

Proposition 5.1

If $cl^*(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy supra dense set in a fuzzy supra submaximal space (X,T^*), then (X,T^*) is a fuzzy supra Baire space .

Proof.

Let (λ_i)'s be fuzzy supra dense sets in a fuzzy supra submaximal space. Then $\lambda_i \in T^*$ in (X,T^*). Now $cl^*(\lambda_i) = 1$ and $int^*(\lambda_i) = \lambda_i$ implies that $cl^* int^*(\lambda_i) = 1$ Then we have $1 - cl^* int^*(\lambda_i) = 0$. This implies that $int^* cl^*(1 - (\lambda_i)) = 0$. Hence $(1-(\lambda_i))$'s are fuzzy supra nowhere dense sets in (X,T^*). Now $cl^*(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, implies that $1-cl^*(\wedge_{i=1}^{\infty}(\lambda_i)) = 0$. Then, $int^*(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where $(1- (\lambda_i))$'s are fuzzy supra nowhere dense sets in (X,T^*). Hence, (X,T^*) is a fuzzy supra Baire space.

Proposition 5.2

If $cl^*(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy supra dense set in a fuzzy supra strongly hyperconnected space (X,T^*) , then (X,T^*) is a fuzzy supra Baire space.

Proof.

Let (λ_i)'s be fuzzy supra dense sets in a fuzzy supra strongly hyperconnected space (X,T^*). By proposition 4.1, Every fuzzy supra strongly hyperconnected space is fuzzy supra submaximal space. Therefore (λ_i) 's be fuzzy supra dense sets in a fuzzy supra submaximal space with $cl^*(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$. Hence by proposition 5.1 , (X,T^*) is a fuzzy supra Baire space

Remark 5.1.

The converse of the above preposition is need not be true. That is, A fuzzy supra topological spaces (X,T*) is a fuzzy supra Baire space then (X,T*) need not be a fuzzy supra strongly hyperconnected space. For consider the example.

Example 5.1.

Let X = {a,b,c}. Then the fuzzy sets λ, μ and γ are defined on X as follows:

λ: X $\rightarrow$[0,1] defined as $\lambda(a) = 1$, $\lambda(b) = 0$, $\lambda(c) = 0.3$.

μ: X $\rightarrow$ [0,1] defined as $\mu(a) = 0.6$, $\mu(b) = 1$, $\mu(c) = 0.7$.

γ: X $\rightarrow$ [0,1] defined as $\gamma(a) = 0.8$, $\gamma(b) = 0$, $\gamma(c) = 1$.

Then, $T^* = \{0, \lambda, \mu, \gamma, \lambda\vee\mu, \mu\vee\gamma, \lambda\wedge\mu, \lambda\vee(\mu\wedge\gamma), \mu\vee(\lambda\vee\gamma), 1\}$, is a fuzzy supra open sets in (X,T^*). The fuzzy sets $1-\lambda$, $1-\gamma$, $1-(\lambda\vee\mu)$, $1- (\mu\vee\gamma)$, $1- (\lambda\wedge\mu)$, $1-[\lambda\vee(\mu\wedge\gamma)]$, $1-[\mu\vee(\lambda\vee\gamma)]$ are fuzzy supra nowheredense sets in (X,T^*). Also, $int^*[(1-\lambda) \vee (1-\gamma) \vee 1-(\lambda\vee\mu) \vee 1- (\mu\vee\gamma) \vee 1- (\lambda\wedge\mu) \vee 1-(\lambda\vee(\mu\wedge\gamma)) \vee 1-(\mu\vee(\lambda\vee\gamma))] = 0$. Therefore (X,T^*) is a fuzzy supra Baire space. But, the fuzzy supra open sets $\{\lambda, \mu, \gamma, \lambda\vee\mu ,\mu\vee\gamma, \lambda\wedge\mu, \lambda\vee(\mu\wedge\gamma), \mu\vee(\lambda\vee\gamma)\}$ are not a fuzzy supra dense sets in (X,T^*). Therefore (X,T^*) is not a fuzzy supra strongly hyperconnected space.

Proposition 5.3

If a fuzzy supra P-space in a fuzzy supra hyper connected space (X, T^*) is a fuzzy supra Baire space.

Proof.

Let α be a fuzzy supra G_δ- set in fuzzy supra P- space in (X,T^*). Since, (X,T^*) is fuzzy supra hyper connected space. Therefore fuzzy supra open set α is a fuzzy supra G_δ- set in (X,T^*). That is, $cl^*(\alpha) =1$. Since α is fuzzy supra dense set in a fuzzy supra P- space. Therefore, $(1-\alpha)$ is a fuzzy supra first category set in (X,T^*). Therefore,$(1-\alpha) = \vee_{i=1}^{\infty}(\alpha_i)$, where (α_i)'s are fuzzy supra nowhere dense sets in (X,T^*). Then $int^*(\vee_{i=1}^{\infty}(\alpha_i)) = int^*(1-\alpha) =1- cl^*(\alpha)=1 - 1 =0$. Hence, $int^* (\vee_{i=1}^{\infty}(\alpha_i)) =0$. Where (α_i)'s are fuzzy supra nowhere dense set in (X,T^*). Then (X,T^*) is fuzzy supra Baire space.

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Fuzzy Setting On Supra σ-Baire Spaces

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1.Introduction

The theory of fuzzy sets was initiated by L.A.Zadeh in his classical paper [8] in the year 1965 as an attempt to develop a mathematically precise framework in which to treat systems of phenomena which cannot themselves be characterized precisely. The potential of fuzzy notion was realized by the researchers and has successfully been applied for investigations in all the branches of Science and Technology. The paper of C.L.Chang [3] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological groups.

The concept of σ −Baire spaces in fuzzy setting was introduced and studied by the authors in [7]. M .E. Abd El- Monsef at al. [1] introduced Fuzzy supra topological spaces generalizing classical supra topological spaces introduced by A.S. Mashhour et al.[4]. In this paper, we introduce the concept of fuzzy supra σ −Baire space by using fuzzy supra σ −nowhere dense sets and investigate several characterizations and examples of fuzzy supra σ −Baire spaces.

2. Preliminaries

Definition 2.1 [5]: A fuzzy topology T on a set X is a family of fuzzy sets in X such that

(1) $0_X, 1_X \in T$;
(2) $A, B \in T \Rightarrow A \wedge B \in T$ and
(3) $A_i \in T \Rightarrow \vee A_i \in T$

The pair (X, T) is called a fuzzy topological space (FTS). The elements of T are called fuzzy open sets and the complement of fuzzy open set is called fuzzy closed set.

Definition 2.2 [2]: The closure and the interior of a fuzzy set A in FTS (X, T) are denoted and defined respectively as

$$Cl(A) = \wedge \{B : A \leq B, B \text{ is a fuzzy closed set in } X\},$$

$$Int(A) = \vee \{B : B \leq A, B \text{ is a fuzzy open set in } X\}.$$

Definition 2.3[1]: A collection T^* of fuzzy open sets in a set X is called a fuzzy supra topology on X if the following conditions are satisfied:

$$(1) 0_X, 1_X \in T \quad and \quad (2) A_i \in T \Rightarrow \vee A_i \in T$$

The pair (X, T^*) is called a fuzzy supra topological space (FSTS). The elements of T^* are called fuzzy supra open sets (FSOS) and the complement of a fuzzy supra open set is called fuzzy supra closed set (FSCS). The collection of all fuzzy supra open sets (resp.fuzzy supra closed sets) of the FSTS (X, T^*) is denoted by FSOS(X) (resp. FSCS(X)).

Remark 2.4 (1):

1. Every FTS is a FSTS.

2. If (X, T^*) is an associated FSTS with the FTS (X, T) (i.e. $T \subseteq T^*$), then every fuzzy open (closed) set in the FTS (X, T) is fuzzy supra open (closed) set in the FSTS (X, T^*).

Definition 2.5 [4]: Let (X, T^*) is a FSTS and A be fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior are denoted and defined respectively as

$$Cl^*(A) = \wedge\{B: A \leq B, B \text{ is a fuzzy supra closed set in } X\},$$

$$Int^*(A) = \vee\{B: B \leq A, B \text{ is a fuzzy supra open set in } X\}.$$

Remark 2.6 [4]:

(1) The fuzzy supra closure of a fuzzy set A in a FSTS is the smallest fuzzy supra closed set containing A.
(2) The fuzzy supra interior of a fuzzy set A in a FSTS is the largest fuzzy supra open set containing in A.
(3) If (X, T^*) is an associated FSTS with the FTS (X, T) and A is any fuzzy set in X, then
$$Int(A) \leq int^*(A) \leq A \leq Cl^*(A) \leq Cl(A).$$

Properties of fuzzy supra closure and fuzzy supra interior which are needed in the sequel, are summarized in the following theorem.

Theorem 2.7 [4]: For any fuzzy sets A and B in a FSTS (X, T^*).

(1) $A \in FSCS(X) \Leftrightarrow Cl^*(A) = A$, $A \in FSOS(X) \Leftrightarrow Int^*(A) = A$;
(2) $A \leq B, \Rightarrow Cl^*(A) \leq Cl^*(B) and\ Int^*(A) \leq Int^*(B)$;
(3) $Cl^*\big(Cl^*(A)\big) = Cl^*(A)\ and\ Int^*\big(Int^*(A)\big) = Int^*(A)$;
(4) $Cl^*(A \vee B) \geq Cl^*(A) \vee Cl^*(B)$;
(5) $Cl^*(A \wedge B) \leq Cl^*(A) \wedge Cl^*(B)$;
(6) $Int^*(A \vee B) \geq Int^*(A) \vee Int^*(B)$;
(7) $Int^*(A \wedge B) \leq Int^*(A) \wedge Int^*(B)$;
(8) $Cl^*(A^C) = \big(Int^*(A)\big)^C, Int^*(A^C) = \big(Cl^*(A)\big)^C$.

3. FUZZY SUPRA σ –NOWHERE DENSE SETS

Definition 3.1: A fuzzy set A in a FSTS (X, T^*) is called fuzzy supra dense if there exists no fuzzy supra closed set B in (X, T^*) such that $A < B < 1$. That is, $Cl^*(A) = 1$.

Definition 3.2: A fuzzy set A in a FSTS (X, T^*) is called fuzzy supra F_σ –set in (X, T^*) if $A = \vee_{i=1}^{\infty} A_i$, where $1 - A_i \in T^*$ for $i \in I$.

Definition 3.3: A fuzzy set A in a FSTS (X, T^*) is called a fuzzy supra G_δ –set in (X, T^*) if $A = \wedge_{i=1}^{\infty} A_i$, where $A_i \in T^*$ for $i \in I$.

Definition 3.4: A fuzzy set A in a FSTS (X, T^*) is called fuzzy supra nowhere dense if there exists no non-zero fuzzy open set B in (X, T^*) such that $B < cl^*(A)$. That is, $int^* cl^*(A) = 0$.

Definition 3.5: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set A in (X, T^*) is called a fuzzy supra σ –nowhere dense set if A is a fuzzy supra F_σ –set in (X, T^*) such that $int^*(A) = 0$.

Example 3.6: Let $X = \{a, b\}$ be a set with a fuzzy supra topology (X, T^*). Then the fuzzy sets $A = \{a_{.6}, b_{.7}\}, B = \{a_{.7}, b_{.8}\}, c = \{a_{.6}, b_{.9}\}$. Then $T^* = \{0, A, B, C, A \vee B, B \vee C, B \wedge C, 1 - A, 1 - B, 1 - C, 1 - A \vee B, 1 - B \vee C, 1 - B \wedge C, 1\}$. Now, consider the fuzzy set $\alpha = (1 - A \vee B) \vee (1 - B \vee C) \vee (1 - B \wedge C)$ in (X, T^*). Then α is fuzzy supra F_σ –set in (X, T^*) and $int^*(A) = 0$ and hence α is a fuzzy supra σ – nowhere dense set in (X, T^*).

Proposition 3.7: In a fuzzy supra topological space (X, T^*) a fuzzy set A is fuzzy supra σ –nowhere dense set in (X, T^*) if and only if $1 - A$ is a fuzzy supra dense and fuzzy supra G_δ –set in (X, T^*).

Proof: Let A be a fuzzy supra σ –nowhere dense set in (X, T^*). Then $A = \vee_{i=1}^{\infty} A_i$, where $1 - A_i \in T^*$ for $i \in I$ and $int^*(A) = 0$. Then $1 - int^*(A) = 1 - 0 = 1$ implies that $cl^*(1 - A) = 1$. Also $(1 - A) = 1 - \vee_{i=1}^{\infty} (A_i) = \wedge_{i=1}^{\infty} (1 - A_i)$ where $(1 - A_i) \in T^*$, for $i \in I$. Hence we have $(1 - A)$ is a fuzzy supra dense and fuzzy supra G_δ –set in (X, T^*).

Conversely, let A be a fuzzy supra dense and fuzzy supra G_δ –set in (X, T^*). Then $A = \wedge_{i=1}^{\infty} (A_i)$ where $(A_i) \in T^*$, for $i \in I$. Now $(1 - A) = 1 - \wedge_{i=1}^{\infty} (A_i) = \vee_{i=1}^{\infty} (1 - A_i)$. Hence $1 - A$ is a fuzzy supra F_σ –set in (X, T^*) and $int^*(1 - A) = 1 - cl^*(A) = 1 - 1 = 0$.[Since A is a fuzzy supra dense]. Therefore $1 - A$ is a fuzzy supra σ –nowhere dense set in (X, T^*).

Proposition 3.8: If A is fuzzy supra dense set in (X, T^*) such that $B \leq (1 - A)$, where B is a fuzzy supra F_σ –set in (X, T^*), then B is a fuzzy supra σ –nowhere dense set in (X, T^*).

Proof: Let A be a fuzzy supra dense set in (X, T^*) such that $B \leq (1 - A)$. Now $B \leq (1 - A)$ implies that $int^*(B) \leq int^*(1 - A) = 1 - cl^*(A) = 1 - 1 = 0$ and hence $int^*(B) = 0$. Therefore B is a fuzzy supra σ –nowhere dense set in (X, T^*).

Proposition 3.9: If A is a fuzzy supra F_σ –set and fuzzy supra nowhere dense set in (X, T^*), then A is a fuzzy supra σ –nowhere dense set in (X, T^*).

Proof: Now $A \leq cl^*(A)$ for any fuzzy set in (X, T^*). Then, $int^*(A) \leq int^* cl^*(A)$. Since A is a fuzzy supra nowhere dense set in (X, T^*), $int^* cl^*(A) = 0$ and hence $int^*(A) = 0$ and A is a fuzzy supra F_σ –set implies that A is a fuzzy supra σ –nowhere dense set in (X, T^*).

Definition 3.10: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set A in (X, T^*) is called fuzzy supra σ –first category if $A = \vee_{i=1}^{\infty} (A_i)$ where (A_i)'s are fuzzy supra σ –nowhere dense sets in (X, T^*). Any other fuzzy set in (X, T^*) is said to be fuzzy supra σ –second category in (X, T^*).

Definition 3.11: Let A be a fuzzy supra σ –first category set in (X, T^*). Then $1 - A$ is called a fuzzy supra σ –residual set in (X, T^*).

Definition 3.12: A fuzzy supra topological space (X, T^*) is called fuzzy supra σ –first category if the fuzzy set 1_X is a fuzzy supra σ –first category space in (X, T^*). That is, $1_X = \vee_{i=1}^{\infty} (A_i)$ where (A_i)'s are fuzzy supra σ –nowhere dense sets in (X, T^*). Otherwise, (X, T^*) will be called a fuzzy supra σ –second category space.

4. FUZZY SUPRA σ –BAIRE SPACE

Definition 4.1: Let (X, T^*) be a fuzzy supra topological space. Then (X, T^*) is called a fuzzy supra σ –Baire Space if $int^*\left(\vee_{i=1}^{\infty} (A_i)\right) = 0$, where (A_i)'s are fuzzy supra σ –nowhere dense sets in (X, T^*).

Example 4.2: Let $X = \{a, b, c\}$ be a set with a fuzzy supra topology (X, T^*). Then the fuzzy sets $A = \{a_{.8}, b_{.6}, c_{.7}\}, B = \{a_{.6}, b_{.9}, c_8\}, c = \{a_{.7}, b_{.5}, c_{.9}\}$. Then $T^* = \{0, A, B, C, A \vee B, A \vee C, B \vee C, B \wedge C, 1 - A, 1 - B, 1 - C, 1 - A \vee B, 1 - B \vee C, 1 - B \wedge C, 1\}$. Now, consider the fuzzy set $\alpha = (1 - A \vee B) \vee (1 - B \vee C) \vee (1 - B \wedge C)$ in (X, T^*). Then α is fuzzy supra F_σ –set in (X, T^*) and $int^*(\alpha) = 0$ and hence α is a fuzzy supra $\sigma -$ nowhere dense set in (X, T^*).

$\beta = (1 - B) \vee (1 - A \vee C) \vee (1 - B \vee (A \wedge C))$ and $int^*(\beta) = 0$ and hence β is a fuzzy supra $\sigma -$ nowhere dense set in (X, T^*). Then α and β are fuzzy supra $\sigma -$ nowhere dense sets in (X, T^*) and also $int^*(\alpha \vee \beta) = 0$ and therefore (X, T^*) is a fuzzy supra σ –Baire Space.

Proposition 4.3: Let (X, T^*) be a fuzzy supra topological space. Then the following are equivalent:

1. (X, T^*) is a fuzzy supra σ –Baire Space.
2. $int^*(A) = 0$ for every fuzzy supra σ –first category set A in (X, T^*).
3. $cl^*(B) = 1$ for every fuzzy supra σ –residual set B in (X, T^*).

Proof: (1) ⇒ (2). Let A be a fuzzy supra σ –first category set in (X, T^*). Then $A = \left(\vee_{i=1}^{\infty} (A_i)\right)$ where (A_i)'s are fuzzy supra σ –nowhere dense sets in (X, T^*). Then, we have $int^*(A) = int^*\left(\vee_{i=1}^{\infty} (A_i)\right)$.

Since (X, T^*) is a fuzzy supra σ –Baire Space, $int^*\left(\vee_{i=1}^{\infty} (A_i)\right) = 0$. Hence $int^*(A) = 0$ for any fuzzy supra σ –first category set A in (X, T^*).

(2) ⇒ (3). Let B be a fuzzy supra σ –residual set in (X, T^*). Then $(1 - B)$ is a fuzzy supra σ –first category set A in (X, T^*). By hypothesis, $int^*(1 - B) = 0$. Then $1 - cl^*(B) = 0$. Hence $cl^*(B) = 1$ for any fuzzy supra σ –residual set B in (X, T^*).

(3) ⇒ (1). Let A be a fuzzy supra σ –first category set in (X, T^*). Then $A = \left(\vee_{i=1}^{\infty} (A_i)\right)$ where (A_i)'s are fuzzy supra σ –nowhere dense sets in (X, T^*). Now A is a fuzzy supra σ –first category set in (X, T^*) implies that $(1 - A)$ is a fuzzy supra σ –residual set in (X, T^*). By hypothesis, we have $cl^*(1 - A) = 1$. Then $1 - int^*(A) = 1$. Hence $int^*(A) = 0$. That

is, $int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$ where (A_i)'s are fuzzy supra σ –nowhere dense sets in (X,T^*). Hence (X,T^*) is a fuzzy supra σ –Baire Space.

Proposition 4.4: If the fuzzy supra topological space (X,T^*) is a fuzzy supra σ –Baire Space, then (X,T^*) is a fuzzy supra σ –second category space.

Proof: Let (X,T^*) is a fuzzy supra σ –Baire Space. Then $int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$ where (A_i)'s are fuzzy supra σ –nowhere dense set in (X,T^*). Then $\vee_{i=1}^{\infty}(A_i)\neq 1_X$ [otherwise, $\vee_{i=1}^{\infty}(A_i)=1_X$ implies that $int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=int^*1_X=1_X$, which implies that $0=1$, a contradiction]. Hence (X,T^*) is a fuzzy supra σ –second category space.

Proposition 4.5: If the fuzzy supra topological space (X,T^*) . If $\wedge_{i=1}^{\infty}(A_i)\neq 0$, where (A_i)'s are fuzzy supra dense and fuzzy supra G_δ –set in (X,T^*), then (X,T^*) is a fuzzy supra σ –second category space.

Proof: Now $\wedge_{i=1}^{\infty}(A_i)\neq 0$ implies that $1-(\wedge_{i=1}^{\infty}(A_i))\neq 1-0=0$. Then we have $\left(\vee_{i=1}^{\infty}(1-A_i)\right)\neq 1$. Since A_i is a fuzzy supra dense and fuzzy supra G_δ –set in (X,T^*), by proposition 3.1, $(1-A_i)$ is a fuzzy supra σ –nowhere dense set in (X,T^*). Hence $\left(\vee_{i=1}^{\infty}(1-A_i)\right)\neq 1$,where $(1-A_i)$'s are fuzzy supra σ –nowhere dense set in (X,T^*). Hence (X,T^*) is not a fuzzy supra σ –first category space. Therefore (X,T^*) is a fuzzy supra σ –second category space.

Proposition 4.6: If the fuzzy supra topological space (X,T^*) is a fuzzy supra σ –first category space, then (X,T^*) is not a fuzzy supra σ –Baire Space.

Proof: Let the fuzzy supra topological space (X,T^*) is a fuzzy supra σ –first category space. Then $\vee_{i=1}^{\infty}(A_i)=1_X$, where (A_i)'s are fuzzy supra σ –nowhere dense sets in (X,T^*).Now $int^*(\vee_{i=1}^{\infty}(A_i))=int^*(1_X)=1\neq 0$. Hence by definition, (X,T^*) is not a fuzzy supra σ –Baire Space.

Proposition 4.7: If the fuzzy supra topological space (X,T^*) is a fuzzy supra σ –Baire Space and if $\vee_{i=1}^{\infty}(A_i)=1$, then there exists atleast one fuzzy supra F_σ –set A_i such that $int^*(A_i)\neq 0$.

Proof: Suppose that $int^*(A_i)=0$, for $i=1\ to\ \infty$, where $(A_i)'s$ are fuzzy supra σ –nowhere dense sets in (X,T^*). Then $\vee_{i=1}^{\infty}(A_i)=1$. Implies that $int^*[\vee_{i=1}^{\infty}(A_i)]=int[1]=1\neq 0$, a contradiction to (X,T^*) being a fuzzy supra σ –Baire Space. Hence $int^*(A_i)\neq 0$, for atleast one fuzzy supra F_σ –set A_i in (X,T^*).

Proposition 4.8: If the fuzzy supra topological space (X,T^*) is a fuzzy supra σ –Baire Space, then no non-zero fuzzy supra open set is a fuzzy supra σ –first category set in (X,T^*).

Proof: Let A be a non-zero fuzzy supra open set in a fuzzy supra σ –Baire Space (X,T^*). Suppose that $A=\vee_{i=1}^{\infty}(A_i)$, where the fuzzy sets (A_i)'s are fuzzy supra σ –nowhere dense sets in (X,T^*). Then $int^*(A_i)=int^*\left(\vee_{i=1}^{\infty}(A_i)\right)$. Since (X,T^*) is a fuzzy supra σ –Baire Space, $int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$. This implies that $int^*(A)=int^*\left(\vee_{i=1}^{\infty}(A_i)\right)$. Since (X,T^*) is a fuzzy supra σ –Baire Space, $int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$. This implies that $int^*(A)=0$. Then we will have $A=0$, which is a contradiction, since $A\in T$ implies that $int^*(A)=A\neq 0$. Hence no non-zero fuzzy supra open set is a fuzzy supra σ –first category set in (X,T^*).

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Supra Soft Regular Open (Closed) Sets On Fuzzy Topological Spaces

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Abstract:

The Main Purpose of this paper is to introduce the concept of fuzzy supra soft topological spaces called fuzzy supra soft regular open (closed)sets. Which is a generalization to the notion of fuzzy soft topological spaces and supra soft topological spaces. We discussed certain properties and relations of fuzzy supra soft regular open sets and fuzzy supra soft regular closed sets in fuzzy supra soft topological spaces are established in the work. Finally, we establish some interesting properties of this notion.

Keywords: Fuzzy supra soft regular open sets, Fuzzy supra soft regular closed sets, Fuzzy supra soft topological spaces.

Introduction

The most appropriate theory for dealing with un-certainties is the theory of fuzzy sets, introduced by Zadeh [16] in 1965. This theory brought a paradigmatic change in mathematics. But, there exists a difficulty, how to set the membership function in each particular case. In 1968, Chang [5] introduced fuzzy topological space, and in 2011, subsequently, Cagman et al. [4] and Shabir et al. [13] introduced soft topological spaces and they defined basic notions of soft topological spaces. In 1999, Molodtsov [8] initiated the concept of soft theory as a new mathematical tool for dealing with uncertainties that are free from the above difficulties. Soft set

theory has rich potential for applications in several directions, few of which had been shown by Molodtsov in his pioneer work. Applications of Soft Set Theory in other disciplines and real-life problems are now catching momentum. Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, theory of probability, theory of measurement, and so on. Maji et.al introduced Fuzzy soft sets [7]. In 2011, Tanay et al. [15] introduced the notion of fuzzy soft topological spaces. In 1937, M. Stone [14] introduced the notion of regular open sets in topological spaces. In 1981, K.K. Azad [3] introduced the concept of a fuzzy regular open set in fuzzy topological spaces. In 1987, M.E. Abd El-Monsef et al. [1] introduced fuzzy supra topological space and defined the basic notion of fuzzy supra topological spaces. In 2019, A.M. Abd El-Latif [2] introduced fuzzy supra soft topological spaces. This paper aims to introduce and study the notion of fuzzy supra soft topological spaces, which is a generalization to the notion of fuzzy soft topological spaces [9]and supra soft topological spaces [6]. It is hoped that our findings will help enhance this study on fuzzy supra soft topological spaces for the researchers.

2. Preliminaries

We start our work with the following definitions, which will be needed in this paper.

Definition2.1[16]

A fuzzy set A in a non-empty set X is characterized by a membership function μ_A: X → [0, 1] = I whose value μ_A (*x*) represents the "degree of membership" of *x* in A for *x* ∈ X. Here, I^X denotes the family of all fuzzy sets on X.

Definition 2.2[8]

Let X be an initial universe and E be a set of parameters. Let P(X) denote the power set of X and A be a non-empty subset of E. A pair (F, A), denoted by F_A, is called a soft set over X, where F is a mapping given by F : A → P(X). For particular e ∈ A, F(e) may be considered the set of e-approximate elements of the soft set (F, A) and if e ∉A, then F(e) = φ i.e F_A = {(e, F(e)) : e ∈ A ⊆ E, F : A → P(X)}. The family of all these soft sets over X is denoted by $SS(X)_A$.

Definition 2.3[7]

Let A ⊆ E. A pair (*f*, A), denoted by f_A, is called a fuzzy soft set over X, where *f* is a mapping given by *f* : A → I^X defined by $f_A(e) = \mu^e_{f_A}$ where $\mu^e_{f_A}$=0 if e∉A and $\mu^e_{f_A} \neq 0$ if e ∈A, where $\overline{0}$ is the membership

function of null fuzzy set over X, which takes value 0 for all $x \in X$ i.e, $\bar{0}(e) = 0 \ \forall \ x \in X$. The family of all these fuzzy soft sets over X denoted by $F\ SS(X)_A$.

Definition 2.4[12]

The complement of a fuzzy soft set (f, A), denoted by $(f, A)^c$, is defined $(f, A)^c = (f^c, A)$ $f^c{}_A: A \to I^X$ is a mapping given by $\mu^e{}_{f^c{}_A} = \bar{1}\mu^e_{f_A} \ \forall \ e \in A$, where $\overline{1}(e) = 1 \ \forall \ x \in X$. clearly $(f^c{}_A)^c = f_A$.

Definition 2.5[7]

A fuzzy soft set f_A over X is said to be a null fuzzy soft set, denoted by $\tilde{0}_A$, if for all $e \in A$, $f_A(e) = \bar{0}$.

Definition 2.6[7]

A fuzzy soft set f_A over X is said to be an absolute fuzzy soft set, denoted by $\tilde{1}_A$, if for all $e \in A$, $f_A(e) = \bar{1}$, where $\bar{1}$ is the membership function of absolute fuzzy set over X, Which takes value 1 for all $x \in X$. Clearly, we have $(\tilde{1}_A)^c = \tilde{0}_A$ and $(\tilde{0}_A)^c = \tilde{1}_A$.

Definition 2.7[7]

Let $f_A, g_B \in F\ SS(X)E$. Then, f_A is fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \subseteq \mu^e_{g_B} \forall\ e \in A$, i.e. $\mu^e_{f_A}(x) = \mu^e_{g_B}(x) \forall\ x \in X$ and $\forall\ e \in A$.

Definition 2.8 [7]

The union of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C$, $h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \vee \mu^e_{g_B} \forall e \in C$. Here, we write $h_C = f_A \sqcup g_B$.

Definition 2.9[7]

The intersection of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cap B$ and for all $e \in C$, $h_C(e) = \mu^{eh}{}_{hC} = \mu^e_{f_A} \wedge \mu^e_{g_B} \ \forall e \in C$. Here, we write $h_C = f_A \sqcap g_B$.

Definition 2.10[15]

Let τ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then τ is called a fuzzy soft topology on X if

(1) $\tilde{1}E, \tilde{0}E \in \tau$, where $\widetilde{0}E(e) = 0$ and $\tilde{1}E(e) = 1, \forall e \in E$,

(2) The union of any members of τ, belongs to τ.

(3) The intersection of any two members of T, belongs to τ.

The triplet (X, τ, E) is called a fuzzy soft topological space over X. Also, each member of τ is called a fuzzy open soft in (X, τ, E). We denote the set of all fuzzy open soft sets by F OS

(X,τ, E), or FOS(X).

Definition 2.11[15]

Let (X, τ, E) be a fuzzy soft topological space. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in, if its relative complement $f^c{}_A$ is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by FCS(X, τ, E), or F CS(X).

Definition 2.12[11]

Let (X, T, E) be a fuzzy soft topological space and $f_A \in$ F SS(X)E. The fuzzy soft closure of f_A, denoted by F cl(f_A) is the intersection of all fuzzy closed soft super sets of f_A. i.e., Fcl(f_A) = $\sqcap\{h_D : h_D$ is fuzzy closed soft set and $f_A \sqsubseteq h_D\}$.

The fuzzy soft interior of g_B, denoted by F int(g_B) is the fuzzy soft union of all fuzzy open soft subsets of g_B. i.e. Fint(g_B) = $\sqcup\{h_D : h_D$ is fuzzy open soft set and $h_D \sqsubseteq g_B\}$.

Definition 2.13 [9]

The fuzzy soft set $f_A \in$ F SS(X)E is called fuzzy soft point if for the element e ∈ E, $\mu^e{}_{f\,A}(x) \neq \bar{0}$ and $\mu^{e'}f_A(x) = \bar{0}$ for each $e' \in E - \{e\}$, and this fuzzy soft point is denoted by $f^e{}_A$.

Definition 2.14 [9]

The fuzzy soft point $f^e{}_A$ is said to be belonging to the fuzzy soft set g_B, denoted by $f^e{}_A \tilde{\in} g_B$, if for the element e ∈ A ∩ B, $\mu^e F_A(x) \leq \mu^e_{g_B}(x)$.

Definition 2.15[12]

Let T be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then T is called a fuzzy supra soft topology on X if

- $\tilde{1}E, \tilde{0}E \in T$, where $\tilde{0}E(e) = 0$ and $\tilde{1}E(e) = 1$, ∀e ∈ E,
- The union of any members of T, belongs to T.

The triplet (X, T, E) is called a fuzzy supra soft topological space (fssts for short) over X. Also, each member of T is called a fuzzy supra open soft in (X, T, E). A fuzzy soft set f_A over X is said to be fuzzy supra closed

soft set in X, if its relative complement $f^c{}_A$ is a fuzzy supra open soft set. We denote the set of all fuzzy supra open (closed) soft sets by F SOS(X) (F SCS(X)).

Definition 2.16[2]

A fuzzy soft set g_B in a fssts (X, T, E) is called fuzzy supra soft neigh- borhood (briefly: fss neighborhood) of the fuzzy soft point $f^e{}_A \tilde{\in} X_E$ if there exists a fuzzy supra open soft set h_C such that $f^e{}_A \tilde{\in} h_C \subseteq g_B$. The fuzzy supra soft neighborhood sys- tem of a fuzzy soft point $f^e{}_A$, denoted by $N\tau(f^e{}_A)$,is the family of all its fuzzy supra soft neighborhoods.

Definition 2.17[2]

Let (X, T, E) be a fssts and Y ⊆ X. Let y_E be a fuzzy soft set over (Y,E) defined by y_E: E→I^Y such that

$y_E(e)=\mu e_{y_E} where\ \mu e_{Y_E}(x) = \begin{cases} 1, & x \in Y, \\ 0, & x \in Y. \end{cases}$

Then, the fssts

$Ty_E = \{y_E \sqcap g_B : g_B \in T\}$ is called the fuzzy supra soft subspace topology for y_E and (Y, Ty_E, E) is called fuzzy supra soft subspace of (X,T,E).

Theorem.1[2]

Let (X, T, E) be a supra soft topological space and f_A, $g_B \in$ SS(X)E. Then

- $Fint^{fs}(\tilde{1}E) = \tilde{1}E$ and $F\ int^{fs}(\tilde{0}E) = \tilde{0}E$
- $F\ cl^{fs}(\tilde{1}E) = 1E$ and $F\ cl^{fs}(\tilde{0}E) = \tilde{0}E$.
- f_A is fuzzy supra open soft if and only if $F\ int^{fs}(f_A) = f_A$.
- $Fint^{fs}(F\ int^{fs}(f_A)) = F\ int^{s}(f_A)$ and $F\ int^{fs}(f_A) \subseteq f_A$.
- f_A is fuzzy supra closed soft set if and only if $Fcl^{fs}(f_A) = f_A$
- $Fcl^{fs}(Fcl^{fs}(f_A)) = F\ cl^{fs}(f_A)$ and $f_A \subseteq F\ cl^{fs}(f_A)$.
- If $f_A \subseteq g_B$, then $Fint^{fs}(f_A) \subseteq Fint^{fs}(g_B)$ and $Fcl^{fs}(f_A) \subseteq (g_B)$
- $Fcl^{fs}(f_A) = [Fint^{s}(f_A{}^c)]^c$.

Theorem.2[2]

Let (X, T, E) be a fssts on X and $f_A \in$ SS(X)E. Then:

- $F\ int^{fs}(f^c{}_A) = \tilde{1}_E - [Fcl^{fs}(f_A)]$.
- $F\ cl^{fs}(f^c{}_A) = \tilde{1}_E - [F\ int^{fs}(f_A)]$.

3. Fuzzy supra soft Regular open (closed) sets

This section aims to introduce the notion of fuzzy supra soft topological spaces. We also introduce some basic definitions of fuzzy supra soft topological spaces and theorems of concept.

Definition 3.1:

Let (F, A) be a fuzzy soft set in a fuzzy supra soft topological space (X, E, T*), then

(F, A) is called

- a fuzzy supra soft regular open set if int*(cl* (F, A)) = (F, A)
- a fuzzy supra soft regular closed set if cl*(int*(F, A)) = (F, A).

Theorem3.2:

Let (F, A) be a fuzzy soft set in a fuzzy supra soft topological space (X, E, T*), then

(F, A) is a fuzzy supra soft regular open set if and only if $(F, A)^c$ is a fuzzy supra soft regular closed set.

Proof:

Let (F, A) be a fuzzy supra soft set then,

$$int^*(cl^*(F, A)) = (F, A)$$

$$[int^* (cl^*(F, A)]^c = (F, A)^c$$

$$cl^*[(cl^* (F, A))^c] = (F,A)^c$$

$$cl^*(int^*[(F, A)^c)]) = (F,A)^c$$

$$cl^* (int^*(F, A)^c) = (F,A)^C$$

(F^C, A) is a fuzzy supra soft regular closed set.

Conversely, Let $(F, A)^c$ be a fuzzy supra soft regular closed set.

Consider,

$$cl^*(int^* ((F, A)^c)) = (F, A)^c$$

$$[cl^*(int^* ((F , A)^c))]^c = [(F, A)^c]^c$$

$$int^* [(int^* ((F, A)^c))^c] = (F, A)$$

$$int^* (cl^* [((F, A)^c)^c]) = (F , A)$$

$$int^* (cl^* (F, A)) = (F , A).$$

Since (F, A) is a fuzzy supra soft regular open set.

Theorem 3.3:

Every fuzzy supra soft regular open set is a fuzzy supra soft open set.

Proof:

Let (F, A) be a fuzzy supra soft regular open set in (X, E, T*). Then int*(cl* (F, A)) = (F, A). By definition of fuzzy supra soft interior, int* (F, A) is the largest fuzzy supra soft open set contained in (F, A). int*(cl*(F, A)) is the largest fuzzy supra soft open set contained in cl*(F, A). (F, A) is a fuzzy supra soft open set.

Theorem 3.4:

Every fuzzy supra soft regular closed set is a fuzzy supra soft closed set.

Proof:

Let (F, A) be a fuzzy supra soft regular closed set (X, E, T*). cl*(int*(F, A)) = (F, A). By the definition of fuzzy supra soft closure, cl*(F, A) is the smallest fuzzy supra soft closed set containing (F, A). cl*(int*(F, A)) is the smallest fuzzy supra soft closed set containing int*(F, A). (F, A) is a fuzzy supra soft closed set.

Theorem 3.5:

The fuzzy supra soft closure of a fuzzy supra soft open set is a fuzzy supra soft regular closed set.

The fuzzy supra soft interior of a fuzzy supra soft closed set is a fuzzy supra soft regular open set.

Proof:

Let (F, A) be a fuzzy supra soft open set in a fuzzy supra soft topological space

(X, E, T*).

By the definition of fuzzy supra soft interior,

$$\text{int}^*(F, A) \leq (F, A)$$

$$\text{Since} \quad \text{int}^*(\text{cl}(F, A)) \leq \text{cl}^*(F, A)$$

$$\text{cl}^*(\text{int}^*(\text{cl}^*(F, A))) \leq \text{cl}^*(\text{cl}^*(F, A))$$

$$\text{cl}^*(\text{int}^*(\text{cl}^*(F, A))) \leq \text{cl}^*(F, A) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\text{since } (F, A) \leq \text{cl}^*(F, A)$$

$$\text{int}^*(F, A) \leq \text{int}^*(\text{cl}^*(F, A))$$

$$(F, A) \leq int^*(cl^* (F, A))$$

$$cl^*(F, A) \leq cl^*(int^*(cl^*(F, A))) \text{ ……..(2)}$$

From (1) & (2)

$$cl^*(int^* (cl^*(F, A))) = cl^* (F, A)$$

Since fuzzy supra soft closure of a fuzzy supra soft open set is a fuzzy supra soft regular closed set.

Let (F, A) be a fuzzy supra soft closed set in a fuzzy supra soft topological space (X, E, T*). By the definition of fuzzy supra soft closure, $(F, A) \leq cl^* (F,A)$.

$$int^*(int^*(F, A) \leq int^*(cl^*(int^*(F, A)))$$

$$int^*(F, A) \leq int^*(cl^*(int^*(F, A))) \text{ ……(3)}$$

Since $int^*(F, A) \leq (F, A)$

$$cl^*(int^*(F, A)) \leq cl^* (F, A)$$

$$cl^*(int^*(F, A)) \leq (F, A)$$

$$int^*(cl^*(int^*(F, A))) \leq int^* (F, A) \text{……(4)}$$

From (3) & (4)

$$int^* (cl^* (int^*(F, A))) = int^*(F, A)$$

Since fuzzy supra soft interior of a fuzzy supra soft closed set is a fuzzy supra soft regular open set.

Definition 3.6:

Let (X, E, T*) be a fuzzy supra soft topological space and (F, A) be a fuzzy soft set in X, then the fuzzy supra soft regular closure and fuzzy supra soft regular interior of (F, A) is denoted and defined respectively as

$$rcl^*(F, A) = \wedge\{(G, B)|(G, B) \text{ is a fuzzy supra soft regular closed set in X and } (F, A) \leq (G, B)\}$$

$$rint^*(F,A) = \vee\{(G,B)|(G,B) \text{ is a fuzzy supra soft regular open set in X and } (G, B) \leq (F, A)\}$$

Remark3.7:

Obviously, rint* (F, A) is a fuzzy supra soft regular open set, and rcl* (F, A) is a fuzzy supra soft regular closed set.

Theorem 3.8:

For any fuzzy soft set (F, A) in a fuzzy supra soft topological space (X, E, T*),

- $[\text{rint}^*(F, A)]^c = \text{r cl}^*(F, A^c)$
- $[\text{r cl}^*(F, A)]^c = \text{r int}^*(F, A^c)$

Proof:

Consider

$\text{r int}^*(F, A) = \{(G, B) |$ (G, B) is a fuzzy supra soft regular open set in X and $(G, B) \leq (F, A)\}$

$[\text{rint}^*(F, A)]^c = 1 - \vee\{(G, B) |$ (G, B) is a fuzzy supra soft regular open set in X and $(g, B) \leq (F, A)\}$

$= \Lambda\{(g, B)^c |$ $(g, B)^c$ is a fuzzy supra soft regular closed set in X and $(g, B)^c \geq (f, A)^c\}$

$= \text{rcl}^*(F, A^c)$

$\text{r cl}^*(F,A) = \Lambda\{(G, B) |$ (G, B) is a fuzzy supra soft regular set in X and $(F, A) \leq (G, B)\}$

$[\text{r cl}^*(F, A)]^c = 1 - \Lambda\{(G, B) |$ (G, B) is a fuzzy supra soft regular closed set in X and $(F, A) \leq (G, B)\}$

$= \vee\{(G, B)^c |$ $(G, B)^c$ is a fuzzy supra soft regular open set in X and $(F, A)^c \geq (G, B)^c\}$

Theorem 3.9:

For any two fuzzy soft sets (F, A) and (G, B) in fuzzy supra soft topological space (X, E, T*), Then if $(F, A) \leq (G, B)$ implies

- $rint^*(F, A) \leq r\,int^*(G, B)$
- $r\,cl^*(F, A) \leq r\,cl^*(G, B)$

Proof:

$rint^*(F, A) = \vee\{(h, c) \mid (h, c)$ is a fuzzy supra soft regular open set in X and $(h, c) \leq (F, A)\}$

since $rint^*(F, A)$ is the largest fuzzy supra soft regular open set contained in (F,A).

$r\,int^*(F, A) \leq (F, A)$

$(F, A) \leq (G, B)$ implies that

$rint^*(F, A) \leq (F, A) \leq (G, B)$

implies $rint^*(F, A) \leq (G, B)$

since $r\,int^*(F, A)$ is the fuzzy supra soft regular open set contained in (G, B).

But $rint^*(G, B)$ is the largest fuzzy supra soft regular open set contained in (G, B).

Since $rint^*(F, A) \leq r\,int^*(G, B)$.

$(F, A) \leq (G, B)$ iff $(F, A)^c \geq (G, B)^C$

Consider $(G, B)^c \leq (F, A)^c$

$rint^*(G, B)^c \leq r\,int^*(F, A^c)$

$(rcl^*(G, B))^c \leq (rcl^*(F, A))^c$

Since $rcl^*(F, A) \leq rcl^*(G, B)$.

Conclusion:

This paper introduces and studies the notion of fuzzy supra soft topological spaces. We introduce some new concepts in fuzzy supra soft topological spaces. Certain properties and relations of fuzzy supra soft regular open sets and fuzzy supra soft regular closed sets in fuzzy supra soft topological spaces are established in the work. In the end, we must say that, this paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application. It is hoped that our findings will help enhancing this study on fuzzy supra soft topological spaces for the researchers.

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On fuzzy Supra Semi–Baire Spaces

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ABSTRACT: In this paper the concepts of fuzzy supra semi-nowhere dense set, fuzzy supra semi-first category, fuzzy supra semi second category, fuzzy supra semi –αbaire space are introduced and the characterizations of fuzzy supra semi-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

KEYWORDS: Fuzzy supra semi-open set, Fuzzy supra semi-nowhere dense set, Fuzzy supra semi- first category, Fuzzy supra semi-second category and Fuzzy supra, semi -Baire spaces.

1. INTRODUCTION

Abd El-Monsef et al. [1] introduced Fuzzy supra Topological space. The natural generalizations of classical supra topological spaces introduced by Mashhour. Due to the importance and need for the fuzzification of weaker forms of classical topological notions, several works has been found in this field among the various existing fuzzy topological spaces in recent years, Fuzzy semi-continuous functions are investigated by Won Keun Min [2]. Moreover, Ahmed et al. [3] worked on fuzzy semi-open and fuzzy semi-closed mapping and established number of characterizations.

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by Zadeh [4]. Chang [5] introduced the theory of fuzzy topological spaces. Since then much attention have been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy Topology has been found to be very useful in solving many practical problems. It has been shown that the fuzzy kahler manifolds, which are based on a topology, play the important role in ε^{∞}theory [6].

The concept of Baire space method studied extensively in classical topology [7-10]. The concept of Baire spaces in fuzzy setting was introduced and studied by Thangaraj and Anjalmose [11]. The aim of this paper is to introduce the concepts of fuzzy supra semi-Baireness in fuzzy topological spaces.

The aim of this paper is to further contribute to the study of fuzzy supra semi-open and fuzzy supra semi-closed sets defined by Sahidul Ahmed and Biman Chandra chetia [12] by establishing several important fundamental identities and inequalities supported by counter examples. We define fuzzy supra semi-nowhere dense set and fuzzy supra semi-baire space and establish some interesting of these spaces.

2. PRELIMINARIES

Now we introduce some Basic notions to be used in the sequel. In this work we shall mean a non- empty set X together with a fuzzy supra topology T* and denote it by (X,T*) .

Definition 2.1[4]: A Fuzzy set in X is a map f: $X \to [0,1] = I$. The family of fuzzy sets of X is denoted by I^X.

Following are some basic operations in fuzzy sets inX. For the fuzzy sets f and g inX,

1) $f = g$ if and only if $f(x) = g(x)$ for all $x \in X$.
2) $f \le g$ if and only if $f(x) \le g(x)$ for all $x \in X$.
3) $(f \vee g)(x) = \max\{f(x), g(x)\}$ for all $x \in X$.
4) $(f \wedge g)(x) = \min\{f(x), g(x)\}$ for all $x \in X$.
5) $f^c(x) = 1 - f(x)$ for all $x \in X$ here f^c denotes the complement of f.
6) For a family $\{f\lambda / \lambda \in \Lambda\}$ of fuzzy sets defined on a set X,
   $(\vee_{\lambda\in\Lambda} f_\lambda)(x) = \vee_{\lambda\in\Lambda}(f_\lambda(x))$
   $(\wedge_{\lambda\in\Lambda} f_\lambda)(x) = \wedge_{\lambda\in\Lambda}(f_\lambda(x))$.
7) For any $\alpha \in I$, the constant fuzzy set α in X is a fuzzy set in X defined by $\alpha(x)=\alpha$ for all $x \in X$.
8) 0 denotes null fuzzy set in X and 1 denotes universal fuzzy set in X.

Definition 2.2[5]: A fuzzy topological space is a pair (X,δ) where X is a nonempty set and δ is a family of 0 fuzzy set on X satisfying the following properties:

1) The constant functions 0 and 1 belong to δ.
2) $f,g \in \delta$ implies $f \wedge g \in \delta$.
3) $f_\lambda \in \delta$ for each $\lambda \in \Lambda$ implies $(\vee_{\lambda\in\vee} f_\lambda) \in \delta$.

Then δ is called a fuzzy topology on X. Every member of δ is called fuzzy open set. The complement of a fuzzy open set called fuzzy closed set.

Definition 2.3[5]: The closure and interior of a fuzzy set $f \in I^x$ are defined respectively as,

$cl(f) = \wedge\{g/g$ is a fuzzy closed set in X and $f \le g\}$

$int(f) = \vee\{g/g$ is a fuzzy open set in X and $g \le f\}$

Clearly cl(f) is the smallest fuzzy closed set containing f and int(f) is the largest fuzzy open set contained in f.

Definition 2.4[11]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non - zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\, cl(\lambda)=0$.

Definition 2.5[11]: Let λ bea fuzzy first category set in a fuzzy topological space (X,T). Then $1 - \lambda$ is called a fuzzy residual set in (X, T).

Definition 2.6[11]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if int

$(\vee_{i=i}^{\infty}(\lambda_i)) = 0$, where λ_is are fuzzy nowhere dense sets in (X,T).

Definition 2.7[12]: Let (X,T) be any fuzzy topological space and λ be any fuzzy set in (X,T).
We define cl(λ) = ∧ {μ/λ ≤ μ,1–μ∈T} and int (λ)=⊕ { μ/μ ≤ λ, μ∈T}.
For any fuzzy set in a fuzzy topological space (X,T) it is easy to see that 1– cl (λ)=int (1–λ) and 1–int(λ)=cl(1–λ).

Definition 2.8[15]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_is are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.9[15]: A fuzzy topological space (X,T) is called fuzzy first category space if $1 = \vee_{i=i}^{\infty}(\lambda_i)$ where (λ_i)s are fuzzy nowhere dense sets in(X,T). A topological space, which is not of fuzzy first category, is said to be of fuzzy second category space.

Definition 2.10[16]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzz α - dense if there exists no fuzzy α - closed set μ in (X,T) such that λ<μ<1.

Definition 2.11[17]: A collection δ^* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied :
1) 0 and 1 belongs to δ^*.
2) $f_\lambda \in \delta^*$ for each $\lambda \in \wedge$ implies $(\vee_{\lambda\in\wedge} f_\lambda) \in \delta^*$.
The pair (X,δ^*) is called a fuzzy supra topological space. The elements of δ^* called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition 2.12[17]: let (X,δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior of f defined respectively as
$cl^*(f) = \wedge \{g/g$ is a fuzzy supra closed set in X and $f \le g\}$
$int^*(f) = v \{g/g$ is a fuzzy supra open set in X and $g \le f\}$.

Definition 2.13[17]: let (X,δ) be a fuzzy topological space and δ^* be a fuzzy supra topology on X. We call δ^* is a fuzzy supra topology associated with δ if $\delta \le \delta^*$.

Remark 2.14[17]:
1) The fuzzy supra closure of a fuzzy set f in a fuzzy supra topological space is the smallest fuzzy supra closed set containing f.
2) The fuzzy supra interior of a fuzzy set f in a fuzzy supra topological space is the largest fuzzy supra open set contained in f.
3) If (X,δ^*) is an associated fuzzy supra topological space with the fuzzy topological space (X,δ) and f is any fuzzy set in X, then $int(f) \le int^*(f) \le f \le cl^*(f) \le cl(f)$.

Theorem 2.15[17]: For any fuzzy sets f and g in a fuzzy supra topological space (X,δ^*),
1) f is a fuzzy supra closed set if and only if $cl^*(f)=f$.
2) f is a fuzzy supra open set if and only if $int^*(f)=(f)$.
3) $f \le g$ implies $int^*(f) \le int^*(g)$ and $cl^*(f) \le cl^*(g)$
4) $cl^*(cl^*(f)) = cl^*(f)$ and $int^*(int^*(f)) = int^*(f)$.
5) $cl^*(f \vee g) \ge cl^*(f) \vee cl^*(g)$
6) $cl^*(f \wedge g) \le cl^*(f) \wedge cl^*(g)$
7) $int^*(f \vee g) \ge int^*(f) \vee int^*(g)$
8) $int^*(f \wedge g) \le int^*(f) \wedge int^*(g)$
9) $cl^*(f^c) = [int^*(f)]^c$
10) $int^*(f^c) = [cl^*(f)]^c$.

Definition 2.16[17]: Let (X,δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra semi open set if, $f \le cl^*(int^*(f))$. The complement of a fuzzy supra semi open set is called a fuzzy supra semi closed set.

Definition 2.17[19]: Let (X, T*) bea fuzzy supra topological space. A fuzzy set f is called fuzzy supra α-open set, if $f \le int^*[cl^*(int^*(f))]$ and a fuzzy set f is called fuzzy supra α - closed set if $cl^*[int^*(cl^*(f))] \le f$.

Definition 2.18: Let (X, T_1^*) and (Y,T_2^*) be two FSTS's. A Mapping $f: X \to Y$ is called
1) fuzzy supra semi-open if f(A) is fuzzy supra open in Y for each A ∈ FSSCS (X),
2) fuzzy supra semi-closed if f(A) is fuzzy supra closed in Y for each A ∈ FSSOS (X).

Theorem 2.19: Let $f:X \to Y$ be a mapping between two FSTS's. Then the following are equivalent:
1) f is fuzzy supra semi-open,
2) $f(sint^*(A)) \le int^*(f(A))$ for each fuzzy set A of X,
3) $sint^*(f^{-1}(B)) \le f^{-1}(int^*(B))$ for each fuzzy set B of Y,
4) $f^{-1}(cl^*(B)) \le scl^*(f^{-1}(B))$ for each fuzzy set B of Y.

3. FUZZY SUPRA SEMI-NOWHERE DENSE SETS

Definition 3.1: Let (X,T*) be a fuzzy supra topological space. A fuzzy set λ in (X,T*) is called a fuzzy supra semi-nowhere dense set if there exists no non-zero fuzzy supra semi -open set μ in (X,T*) Such that $\mu < scl^*(\lambda)$. That is, $sint^*scl^*(\lambda)=0$.

Example 3.1: Let X={a, b, c}. The fuzzy sets λ,μ, γ, α and β are defined on X as follows:
λ:X →[0,1] define as λ (a)=0.7; λ (b)=0.3; λ (c)=0.5,
μ:X→[0,1] define as μ (a)=0.2; μ (b)=0.8; μ (c)=0.6,
γ:X→[0,1] define as γ(a)=0.5; γ(b)=0.7; γ(c)=0.8,
α:X→[0,1] define as α (a)=0.9; α (b)=0.8; α (c)=0.7,
β: X →[0,1] define as β(a)=0,9; β(b)=0.9; β(c)=0.6.
Then T = {0, λ, μ, γ,(λvμ), (λvγ), (μvγ), (λ∧μ), (λ∧γ),(μ∧γ),λvμv λ, μv (λ∧ γ),
λv (μv γ), γ∧ (λvμ), 1} is a fuzzy topology on X. The non-zero fuzzy supra semi- open sets
in (X, T*) are λ, μ, γ, (λvμ), (λv γ),), (μvγ), (λ∧μ), (λ∧γ), (μ∧γ), λvμv λ, μv(λ∧ γ),
λv(μvγ), γ∧(λvμ), Now the fuzzy sets 1–λ, 1 –μ, 1 –γ, 1 –(λvμ), 1 –(λvγ),1– λ∧γ,
1 –(μvγ), 1 –λv (μ∧γ), 1 –(μv (λ∧ γ)), 1 –λvμvγ,1 –γ∧λvμ, 1 –α, 1 –β are fuzzy
supra semi-nowhere dense sets in (X,T*).

Proposition 3.1: The complement of a fuzzy suprasemi-nowhere dense set in a fuzzy topological space (X,T*) need not be fuzzy supra semi-nowhere dense set.

Proof: For example 3.1,(1–α) is a fuzzy supra semi-nowhere dense set in (X,T*) where
as α=1–(1–α) is not a fuzzy supra semi-nowhere dense set in (X,T*).

Proposition3.2: If λ and μ are fuzzy supra semi-nowhere dense set in a fuzzy topological
space (X,T*), then λ∨μ need not be fuzzy supra semi-nowhere dense set in (X,T*).
Proof: For, in Example 3.1, 1–α, 1–λ are fuzzy supra semi-nowhere dense set in (X,T*).
But (1–α) ∨ (1–λ)= δimplies that sint*scl*(δ)≠0. Therefore union of fuzzy supra semi -
nowhere dense sets need not be fuzzy supra semi-nowhere dense set in(X,T*).

Proposition3.3: If the fuzzy sets λand μ are fuzzy supra semi-nowhere dense sets in a
Fuzzy topological space (X,T*) then λ∧μ is a fuzzy suprasemi -nowhere dense set
in(X,T*).
Proof: Let the fuzzy sets λ and μ be fuzzy upra semi-nowhere dense set in (X,T*). Now
sint*scl*(λ∧μ)≤ sint*scl*(λ)∧ sint*scl*
(μ)≤0∧0 (sincesint* scl*(λ)=0 and
sint*Scl*(μ)=0. That is, sint*scl*(λ∧μ)=0. Hence (λ∧μ) is a fuzzy supra semi-nowhere
dense set in (X,T*).

Proposition3.4: If λ is a fuzzy supra semi-nowhere dense sets in a fuzzy topological space
(X,T*) then Sint*(λ)=0.
Proof: Letλbe a fuzzy supra semi-nowhere dense set in (X,T*). Then we have sint*
scl*(λ)=0. Now λ≤ scl*(λ) we have sint*(λ) ≤sint*scl*(λ)=0. Hence sint*(λ)=0.

Proposition3.5: Ifλ is a fuzzy nowhere dense set in fuzzy topological space (X,T*) then int*scl*(λ)=0.
Proof: Let λbe a fuzzy nowhere dense set in (X,T*). Then, we have int* cl*(λ)=0 and int*(λ)=0. Now Scl*(λ)=λ. Since λ is fuzzy supra semi-closed set in (X,T*) [15]implies that int* scl*(λ)= int*(λ)=0. Hence int* scl*(λ)= 0.

Proposition3.6: If λ is a fuzzy supra semi-nowhere dense set and μ is any fuzzy set in a fuzzy topological space (X,T*), then (λ∧μ)is a fuzzy supra semi-nowhere dense set in(X,T*).
Proof: Let λbe a fuzzy supra semi-nowhere dense set in (X,T*).
Then sint* scl*(λ)=0. Now sint* scl*(λ∧μ)≤sint* scl*(λ)∧sint*scl*(μ)≤0∧sint*scl*(μ)=0.
That is , sint*scl*(λ∧μ)=0,Hence (λ∧μ) is a fuzzy supra semi-nowhere dense set in (X,T*).

Definition 3.2[11]: A fuzzy set λ in a fuzzy topological space (X,T*) is called fuzzy supra semi-dense if there exists no fuzzy semi-closed set μ in (X,T*) such that λ<μ<1. That is scl*(λ)=1.
Proposition 3.7: If λis a fuzzy supra semi-dense and fuzzy supra semi-open set in a fuzzy topological space(X,T*) and if μ≤1-λ, then μ is afuzzy supra semi-nowhere dense set in (X,T*).
Proof: Let λ be a fuzzy upra semi- dense set in (X,T*). Then we have Scl*(λ)=1 and sint*(λ)=λ.Now μ≤1–λ,implies thatScl*(μ)≤ Scl*(1–λ).Then Scl*(μ) ≤1–sint*(λ)=1–λ.
Hence Scl*(μ) ≤ scl* (1–λ), which implies that sint*scl*(μ)≤sint*(1–λ)=1–scl*(λ)=1–1=0.
That is, sint* scl*(μ)=0. Hence μ is a fuzzy supra semi-nowhere dense set in (X,T*).

Proposition3.8: If λis a fuzzy supra semi-nowhere dense set in a fuzzy topological space(X,T*). Then 1-λ, is a fuzzy supra semi- dense set in (X,T*).
Proof: Let λ be a fuzzy supra semi-nowhere dense set in (X,T*). Then, sint*scl*(λ)=0Now λ≤scl*(λ) implies that sint*(λ)≤sint*scl*(λ)=0. Then sint*(λ)=0 and scl*(1–λ)=1–sint*(λ)=1–0=1 and hence 1–λ is a fuzzy supra semi-dense set in (X,T*).

4. FUZZY SUPRA SEMI-BAIRE SPACES

Definition 4.1: Let (X,T*) be a fuzzy topological space. Then (X,T*) is called a fuzzy supra semi-Baire space if sint*[$\bigvee_{i=1}^{\infty}(\lambda_i)$]=0, where λ_is are fuzzy supra semi-nowhere dense sets in (X,T*).

Proposition 4.1: If sint* ($\bigvee_{i=1}^{\infty}(\lambda_i)$)=0 where sint* ($\lambda_i$)=0 and λ_is are fuzzy supra semi-closed sets in a fuzzy topological space (X,T*), then (X,T*) is a fuzzy supra semi-Baire space.
Proof: Let λ_is be fuzzy supra semi-closed sets in (X,T*). since sint* (λ_i)=0, By theorem 3.13, the λ_is are fuzzy supra semi-nowhere dense sets in (X,T*), therefore we have sint*($\bigvee_{i=1}^{\infty}(\lambda_i)$)=0, where λ_is are fuzzy supra semi-nowhere dense sets in (X,T*). Hence (X,T*) is a fuzzy supra semi-Baire space.

Proposition 4.2: If scl*($\bigwedge_{i=1}^{\infty}(\lambda_i)$)=1, where λ_is are fuzzy supra semi-nowhere dense and fuzzy supra semi-open sets in a fuzzy topological space (X,T*), then (X,T*) is a fuzzy supra semi-Baire space.
Proof: Nowscl*($\bigwedge_{i=1}^{\infty}(\lambda_i)$)=1implies that1– scl*($\bigwedge_{i=1}^{\infty}(\lambda_i)$)=0. Then we havesint*(1–$\bigwedge_{i=1}^{\infty}(\lambda_i)$)=0, which implies that sint*(($\bigvee_{i=1}^{\infty}(1-\lambda_i)$)=0. Since λ_is are fuzzy supra semi- dense sets in (X,T*), scl*(λ_i)=1 and sint*(1 − λ_i)=1−scl*(λ_i) =1−1=0. Hence we have sint*($\bigvee_{i=1}^{\infty}(1-\lambda_i)$)=0, where sint*(1 − λ_i)=0 and (1 − λ_i)s are fuzzy supra semi-closed sets in (X,T*). Then, by proposition 4.1, (X,T*) is a fuzzy supra semi-Baire space.

Proposition 4.3: Let (X,T*) be a fuzzy topological space. Then the following are equivalent:
1) (X,T*) is a fuzzy supra semi-Baire space.
2) sint*(λ)=0 for every fuzzy supra semi-first category set λ in (X,T*).
3) scl*(μ)=1 for every fuzzy supra semi-residual set in (X,T*).

Proof: (1)→ (2).Let λ be a fuzzy supra semi- first category set in (X,T*). Then λ=$\bigvee_{i=1}^{\infty}(\lambda_i)$ where λ_is are fuzzy supra semi-nowhere dense set in (X,T*). Now sint*(λ)=sint*($\bigvee_{i=1}^{\infty}(\lambda_i)$)=0 (since (X,T*) is a fuzzy supra semi-Baire space).
Therefore sint*(λ)=0.
(2)→ (3).Let μ be a fuzzy supra semi-residual set in (X,T*). Then 1-μ is a fuzzy supra semi-first category set in (X,T*). By hypothesis, sint*(1 –μ)=0. Which implies that 1 –scl* (μ)=0
Hence we have scl*(μ)=1.
(3)→ (1).Let λ be a fuzzy supra semi-first category set in (X,T*), Then λ=$\bigvee_{i=1}^{\infty}(\lambda_i)$ where λ_i are fuzzy supra semi-

nowhere dense set in (X,T*). 1–λ is a fuzzy supra semi-residual set in (X,T*). Since λis a fuzzy supra semi-first category set in (X,T*). By hypothesis, we have scl*(1–λ)=1. Then 1-sint*(λ)=1, which implies that sint*(λ)=0. Hence sint* $(V_{i=1}^{\infty}(\lambda_i))=0$ where λ_is are fuzzy supra semi-nowhere dense sets in (X,T*). Hence (X,T*) is a fuzzy supra semi-Baire space.

Proposition 4.4: If a fuzzy topological space (X,T*) is a fuzzy supra semi-Baire space, then (X,T*) is a fuzzy supra semi-second category space.

Proof: Let (X,T*) be a fuzzy supra semi-Baire space. Then $sint^*(V_{i=1}^{\infty}(\lambda_i))=0$. where λ_i, sare fuzzy supra semi-nowhere dense sets in (X,T*). Then $(V_{i=1}^{\infty}(\lambda_i))\neq 1x$.(Suppose, $(V_{i=1}^{\infty}(\lambda_i))\neq 1x$ implies that $sint^*(V_{i=1}^{\infty}(\lambda_i))=sint^*(1x)$ which implies that 0 = 1, a contradiction). Hence (X,T*) is a fuzzy supra semi-second category space.

Proposition 4.5: If a fuzzy topological space (X,T*) is a fuzzy supra semi-Baire space, Then no non-zero fuzzy supra semi-open sets in (X,T*) is a fuzzy supra semi-first category set in (X,T*).

Proof: suppose that λ is a non-zero fuzzy supra semi-open set in (X,T*) such that $\lambda=V_{i=1}^{\infty}(\lambda_i)$, where λ_is are fuzzy supra semi-nowhere dense set in (X,T*). Then we have $sint^*(\lambda)=sint^*(V_{i=1}^{\infty}(\lambda_i))$.since λ is a non- zero fuzzy supra semi-open set in (X,T*). $sint^*(\lambda)=\lambda$, Then $sint^*(V_{i=1}^{\infty}(\lambda_i))=\lambda\neq 0$. But this is a contradiction to (X,T*) being a fuzzy supra semi-Baire space in which $sint^*(V_{i=1}^{\infty}(\lambda_i))=0$. where λ_is are fuzzy suprasemi-nowhere dense set in (X,T*). Hence, we must have $\lambda\neq(V_{i=1}^{\infty}(\lambda_i))$. Therefore no non-zero fuzzy supra semi-open set in (X,T*) is a fuzzy supra semi-first category set in (X,T*).

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On Fuzzy Semi-Connected Spaces

Eswaran Poongothai, M. Nalini

Abstract. In this paper, we are going to explore the idea of Fuzzy semi-connected spaces and Fuzzy semi-separated sets which are solved by using Fuzzy semi-open and Fuzzy semi-closed sets. In addition, we classify their relations between Fuzzy semi-boundary and Fuzzy semi-irresolute function.

Keywords: Fuzzy semi-connected spaces, Fuzzy semi-disconnected spaces, Fuzzy semi-separated sets and Fuzzy semi-irresolute function.

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1. Introduction

The theory of fuzzy set was introduced by L.A.Zadeh [7] in 1965. Thereafter the paper C.L.Chang [5]in 1968 proved the way for the fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. The author N.Levine [6] introduced the semi-open sets in 1963 and also K.K.Azad[2] introduced and studied the above concept in fuzzy setting. However the notion of connectedness was introduced by Cantor in 1883 and Jordan in 1893.

In this paper, we study fuzzy semi-connected and fuzzy semi-disconnected spaces. Further, we give some characterizations and examples related to this concepts. Also, we analyse fuzzy semi-separated sets and several related examples are studied.

2. **Preliminaries**

Now,we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X.

Definition 2.1. [3]Let λ and μ be any two fuzzy sets in (X, T). Then we define
$\lambda \vee \mu : X \to [o, 1]$ as follows : $(\lambda \vee \mu)(x) = max\{\lambda(x), \mu(x)\}$
$\lambda \wedge \mu : X \to [o, 1]$ as follows : $(\lambda \wedge \mu)(x) = max\{\lambda(x), \mu(x)\}$

Definition 2.2. [3] Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$

Lemma 2.3. [3] *For a family of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X, T), $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case A is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$.*

Definition 2.4. [4] A fuzzy set λ in a fuzzy topological space X is called fuzzy semi-open if $\lambda \leq cl\ int(\lambda)$ and fuzzy semi-closed if $int\ cl(\lambda) \leq \lambda$.

Definition 2.5. [6] Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). We define the fuzzy semi-closure and the fuzzy semi-interior of λ as follows:

(1) $scl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu\ \epsilon\ T\}$ is fuzzy semi-closed set of X.

(2) $sint(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu\ \epsilon\ T\}$ is fuzzy semi-open set of X.

Definition 2.6. [6] A fuzzy set λ i a fuzzy topological space (X, T) is called a fuzzy semi-closed set if $\lambda = scl(\lambda)$ and fuzzy semi-open set if $\lambda = sint(\lambda)$.

Lemma 2.7. [6] *Let λ be a fuzzy set of a fuzzy topological space (X, T). Then*

(1) $1 - scl(\lambda) = sint(1 - \lambda)$

(2) $1 - sint(\lambda) = scl(1 - \lambda)$

Lemma 2.8. [4] *Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let f be a function from the fuzzy topological space (X, T) to the fuzzy topological space (Y, S). Let λ*

be a fuzzy set in (Y, S)*. The inverse image of* λ *under* f *written as* $f^{-1}(\lambda)$ *is the fuzzy set in* (X, T) *defined by* $f^{-1}(\lambda)(X) = \lambda(f(X))$ *for all* $x \in X$*. Also the image of* λ *in* (X, T) *under* f *written as* $f(\lambda)$ *is the fuzzy set in* (Y, S) *defined by*

$$f(\lambda)(y) = \begin{cases} sup\ \lambda\ (x) \\ x\ \in\ f^{-1}(y) \quad if\ f^{-1}(y) is\ non\text{-}empty; \\ \\ 0\ otherwise \end{cases}$$

for each $y \in Y$*. let* $f : (X, T) \to (Y, S)$ *be a mapping. For fuzzy sets* λ *and* μ *of* (X, T) *and* (Y, S) *respectively, the following statements hold.*

- $ff^{-1}(\mu) \leq \mu$;
- $f^{-1}f(\lambda) \geq \lambda$;
- $f(1 - \lambda) \geq 1 - f(\lambda)$;
- $f^{-1}(1\mu) = 1 - f^{-1}f(\mu)$;
- *If* f *is injective, then* $f^{-1}f(\lambda) = \lambda$;
- *If* f *is surjective, then* $ff^{-1}(\mu) = \mu$;
- *If* f *if bijective, then* $f(1 - \lambda) = 1 - f(\lambda)$.

Lemma 2.9. [2] *Let* (X, T) *be a fuzzy topological space. Then for any two fuzzy sets* λ *and* μ*,*

(i) λ *is a fuzzy semi-closed set in* $X \Leftrightarrow Scl(\lambda) = \lambda$.

(ii) $Scl(Scl(\lambda)) = Scl(\lambda)$

(iii) *If* $\lambda \leq \mu$*, then* $Scl(\lambda) \leq Scl(\mu)$.

3. Fuzzy Semi-Connected Spaces

This section deals with fuzzy semi-connected spaces and fuzzy semi-disconnected spaces using fuzzy semi-open and fuzzy semi-closed sets. Furthermore, We give some characterizations related to this concepts.

Definition 3.1. Let (C, ρ) be a fuzzy topological space and λ be the fuzzy set. If there exist two fuzzy semi-open sets μ and γ such that $\lambda = \mu \vee \gamma$ and $\mu \wedge \gamma = 0$, then the fuzzy set λ is called a fuzzy semi-disconnected set in (C, ρ). If there does not exist such two fuzzy semi-open sets, then the fuzzy set λ is called a fuzzy semi-connected set in (C, ρ).

Example 3.2. Let $C = \{a, b\}$ with $\rho = \{0, \lambda, \mu, \gamma, 1\}$ where
$\lambda = \langle (a, 1, 0.3, 0.3), (b, 0.4, 0.6, 0.5) \rangle$ $\mu = \langle (a, 0, 0.3, 1), (b, 0.4, 0.6, 0.5) \rangle$ and
$\gamma = \langle (a, 1, 0, 0.3), (b, 0, 0.6, 1) \rangle$. Then λ, μ and γ are fuzzy semi-open set in (C, ρ). Now, $\mu \wedge \gamma = \langle (a, 0, 0, 1), (b, 0, 0, 1) \rangle = 0$ and $\lambda = \mu \vee v$. Hence by Definition 3.1, λ is a fuzzy semi-disconnected set.

Example 3.3. Let $C = \{a, b\}$ with $\rho = \{0, \lambda, \mu, \gamma, \alpha, 1\}$ where
$\lambda = \langle (a, 0.6, 0.4, 0.2), (b, 0.3, 0.5, 0.4) \rangle$,$\mu = \langle (a, 0.5, 0.3, 0.4), (b, 0.6, 0.2, 0.5) \rangle$ and
$\gamma = \langle (a, 0.4, 0.1, 0.7), (b, 0.8, 0, 0.5) \rangle$ and $\alpha = \langle (a, 0.6, 0.4, 0.2), (b, 0.8, 0.5, 0.4) \rangle$. Now
$\mu \wedge \gamma = \langle (a, 0.4, 0.1, 0.7), (b, 0.6, 0, 0.5) \rangle \neq 0$ and
$\lambda \neq \mu \vee \gamma = \langle (a, 0.5, 0.3, 0.4), (b, 0.8, 0.2, 0.5) \rangle$. Hence by Definition 3.1, λ is a fuzzy semi-connected set.

Definition 3.4. A fuzzy function $f : X \to Y$ is called a fuzzy irresolute function if the inverse image of every fuzzy semi-open set in Y is a fuzzy semi-open set.

Proposition 3.5. *Let X and Y be two fuzzy topological spaces, λ be the fuzzy set in C and $f : X \to Y$ be fuzzy semi-irresolute. If λ is fuzzy semi-connected, then $f(\lambda)$ is fuzzy semi-connected.*

Proof : Suppose $S(\lambda)$ is a fuzzy semi-open sets μ and γ such that $f(\lambda) = \mu \vee \gamma$ *and* $\mu \wedge \gamma = 0$. Thus $\lambda \leq f^{-1}(f(\lambda)) = f^{-1}(\mu \vee \gamma) = f^{-1}(\mu) \vee f^{-1}(\gamma)$ and $f^{-1}(\mu) \wedge f^{-1}(\gamma) = f^{-1}(\mu \wedge \gamma) = f^{-1}(0) = 0$. That is $\lambda = f^{-1}(\mu) \vee f^{-1}(\gamma)$ and $f^{-1}(\mu) \wedge f^{-1}(\gamma) = 0$. That is a contradiction to our assumption. Hence $f(\lambda)$ is a fuzzy semi-connected set.

Definition 3.6. A fuzzy topological space (C, ρ) is fuzzy semi-disconnected space if there exists fuzzy semi-open sets λ, μ in C, $\lambda \neq 0$, $\mu \neq 0$, such that $\lambda \vee \mu = 1$ and $\lambda \wedge \mu = 0$. If

(C, ρ) is not a fuzzy semi-disconnected space, then it is said to be a fuzzy semi-connected space.

Example 3.7. Let $C = \{a, b, c\}$ with $\rho = \{0, \lambda, \mu, 1\}$ where
$\lambda = \langle (a, 1, 1, 0), (b, 0, 0, 1), (c, 0, 0, 0) \rangle$, $\mu = \langle (a, 0, 0, 1), (b, 1, 1, 0), (c, 1, 1, 0) \rangle$ This implies
$\lambda \neq 0, \mu \neq 0$, $\lambda \vee \mu = \langle (a, 1, 1, 0), (b, 1, 1, 0), (c, 1, 1, 0) \rangle = 1$ and
$\lambda \wedge \mu = \langle (a, 0, 0, 1), (b, 0, 0, 1), (c, 0, 0, 1) \rangle = 0$ where λ and μ are fuzzy open sets in (C, ρ). Since every fuzzy open set is a fuzzy semi-open set, λ and μ are fuzzy semi-open sets. hence (C, ρ) is a fuzzy semi-disconnected space.

Example 3.8. Let $C = \{a\}$ with $\rho = \{0, \lambda, \mu, 1\}$ where $\lambda = \langle (a, 0.3, 0.5, 0.2) \rangle$,
$\mu = \langle (a, 0.2, 0.4, 0.5) \rangle$ This implies $\lambda \neq 0, \mu \neq 0$, $\lambda \vee \mu = \lambda \neq 1$ and $\lambda \wedge \mu = \mu \neq 0$ where λ and μ are fuzzy open sets in (C, ρ). Since every fuzzy open sets is a fuzzy semi-open set, λ and μ are fuzzy semi-open sets. Hence (C, ρ) is fuzzy semi-connected space.

Proposition 3.9. *Let (C, ρ) be a fuzzy semi-connected space and $\sigma \leq C$. Then (C, σ) is a fuzzy semi-connected space.*

Proof : Suppose (C, ρ) is a fuzzy semi-disconnected space. Then By Definition 3.6, there exist two fuzzy semi-open sets λ, μ in σ, $\lambda \neq 0, \mu \neq 0$ such that $\lambda \vee \mu = 1$ and $\lambda \wedge \mu = 0$. Given that $\sigma \leq C$. Thus λ, μ in C, $\lambda \neq 0, \mu \neq 0$ such that $\lambda \vee \mu = 1$ and $\lambda \wedge \mu = 0$. That is (C, ρ) be a fuzzy semi-disconnected space. That is a contradiction to our hypothesis. Hence (C, σ) is a fuzzy semi-connected.

Proposition 3.10. *Let X and Y be two fuzzy topological spaces and $f : X \to Y$ be the fuzzy semi-irresolute and surjective mapping. If (C, ρ) is fuzzy semi-connected (Y, S) is fuzzy semi-connected.*

Proof : Suppose (Y, S) is fuzzy semi-disconnected. Then there exist two fuzzy semi-open sets μ and γ such that $\mu \vee \gamma = 1$ and $\mu \wedge \gamma = 0$. Thus $f^{-1}(\mu) \vee f^{-1}(\gamma) = f^{-1}(\mu \vee \gamma) = f^{-1}(1) = 1$ and $f^{-1}(\mu) \wedge f^{-1}(\gamma) = f^{-1}(\mu \wedge \gamma) = f^{-1}(0) = 0$ Hence (Y, S) is a fuzzy semi-connected space.

Proposition 3.11. *For a fuzzy topological space (C, ρ) the following conditions are equivalent:*

(i) *C is a fuzzy semi-connected space.*

(ii) *The only fuzzy sets of 1 which are both fuzzy semi-open and fuzzy semi-closed are 0 and 1.*

Proof : (i) $\Rightarrow$ (ii) Let C be a fuzzy semi-connected space. Let λ be a fuzzy set of C which is both fuzzy semi-open and fuzzy semi-closed. Then λ and $c(\lambda)$ are disjoint fuzzy semi-open sets and $1 = \lambda \vee c(\lambda)$. Since C is fuzzy semi-connected, either $\lambda = 0$ (or) $c(\lambda) = 0$. That is, either $\lambda = 0$ or $\lambda = 1$.

(ii) $\Rightarrow$ (i) Suppose (C, ρ) is a fuzzy semi-disconnected space. Then there exist two disjoint non-empty semi-open sets λ and μ such that $1 = \lambda \vee \mu$. Since $\lambda = c(\lambda)$, λ is fuzzy semi-closed. By our assumption $\lambda = 0$ or $\lambda = 1$ which is a contradiction to λ is a non-empty fuzzy semi-open set. Therefore (C, ρ) is fuzzy semi-connected.

4. Fuzzy Semi-Separated Sets

In this section, we investigate certain properties between fuzzy semi-connected spaces and fuzzy semi-separated sets with suitable examples.

Definition 4.1. Two non-empty fuzzy sets λ and μ in (C, ρ) called fuzzy semi-separated sets if $FScl(\lambda) \wedge \mu = \lambda \wedge FScl(\mu) = 0$.

Proposition 4.2. *Any two disjoint non-empty fuzzy semi-closed sets are fuzzy semi-separated.*

Proof : Let λ and μ be two disjoint non-empty fuzzy semi-closed sets.

Then $FScl(\lambda) \wedge \mu = \lambda \wedge FScl(\mu) = \lambda \wedge \mu = 0$. Thus λ and μ are fuzzy semi-separated. Also, fuzzy semi-separated sets are not fuzzy semi-closed.

Example 4.3. Let $C = \{a, b\}$ with $\rho = \{0, \lambda, \mu, \gamma, \alpha, 1\}$ and $c(\rho) = \{1, c(\lambda), c(\mu),$
$c(\gamma), c(\alpha), 0\}$ where $\lambda = \langle (a, 0.5, 1, 0.3), (b, 0.6, 1, 0.3)\rangle, B = \langle (a, 0, 0.2, 1), (b, 0, 0, 1)\rangle,$
$C = \langle (a, 1, 0.7, 0.), (b, 1, 0.5, 0)\rangle, D = \langle (a, 0.5, 0.7, 0.3), (b, 0.6, 0.5, 0.3)\rangle,$

$c(\lambda) = \langle(a, 0.3, 0, 0.5), (b, 0.3, 0, 0.6)\rangle, c(\mu) = \langle(a, 1, 0.8, 0), (b, 1, 1, 0)\rangle,$

$c(\gamma) = \langle(a, 0, 0.3, 1), (b, 0, 0.5, 1)\rangle, c(\alpha) = \langle(a, 0.3, 0.3, 0.5), (b, 0.3, 0.5, 0.6)\rangle.$

Then $FScl(c(\lambda)) = c(\lambda)$ and $FScl(\mu) = c(\gamma)$. Thus $FScl(c(\lambda)) \wedge \mu = c(\lambda) \wedge \mu = 0$ and $c(\lambda) \wedge FScl(\mu) = c(\lambda) \wedge c(\gamma) = 0$. Therefore $c(\lambda)$ and μ are fuzzy semi-separated sets. But μ is not a fuzzy semi-closed set.

Proposition 4.4. (i) *If λ and μ are fuzzy semi-separated and $\gamma \leq \lambda$,$\alpha \leq \mu$, then γ and α are also fuzzy semi-separated.*

(ii) *If λ and μ are fuzzy semi-open and if $H = \lambda \wedge \gamma(\mu)$ and $G = \mu \wedge (\lambda)$, then H and G are fuzzy semi-separated.*

Proof : (i) Let λ and μ be fuzzy semi-separated sets. Then

$FScl(\lambda) \wedge \mu = 0 = \lambda \wedge FScl(\mu)$. Since $\gamma \leq \lambda$, $\alpha \leq \mu$ and by Lemma 2.9 (iii),

$FScl(\gamma) \leq FScl(\lambda)$ and $FScl(\alpha) \leq FScl(\mu)$. Thus $FScl(\gamma) \wedge \alpha \leq FScl(\lambda) \wedge \mu = 0$. Hence $FScl(\gamma) \wedge \alpha = 0$. Similarly $FScl(\alpha) \wedge \gamma \leq FScl(\mu) \wedge \lambda = 0$. Hence $FScl(\alpha) \wedge \gamma = 0$.

Therefore γ and α are fuzzy semi-separated sets.

(ii) Let λ and μ be both fuzzy semi-open in X. Then $\gamma(\lambda)$ and $\gamma(\mu)$ are fuzzy semi-closed. Since $H \leq \gamma(\mu)$ and by Lemma 2.9 (iii), $FScl(H) \leq FScl(\gamma(\mu))) = \gamma(\mu)$. Thus $FScl(H) \wedge \mu = 0$. Since $G \leq \mu$. $FScl(H) \wedge G \leq FScl(H) \wedge \mu = 0$. Thus $FScl(H) \wedge G = 0$. Similarly $FScl(G) \wedge H = 0$. Hence H and G are fuzzy semi-separated sets.

Proposition 4.5. *The fuzzy sets λ and μ in (C, ρ) are fuzzy semi-separated if and only if there exist fuzzy semi-open U and V in (C, ρ) such that $\lambda \leq U$, $\mu \leq V$ and $\lambda \wedge V = 0$ and $\mu \wedge U = 0$*

Proof : Let λ and μ be fuzzy semi-separated. Then $\lambda \wedge FScl(\mu) = 0 = FScl(\lambda) \wedge \mu$. Let $V = \gamma(FScl(\lambda))$ and $U = \gamma(FScl(\mu))$. Then U and V are fuzzy semi-open sets such that $\lambda \leq U$, $\mu \leq V$ and $\lambda \wedge V = 0$ and $\mu \wedge U = 0$.

Conversely, let U and V be fuzzy semi-open set such that $\lambda \leq U$, $\mu \leq V$ and $\lambda \wedge V = 0$, $\mu \wedge U = 0$. Then $\lambda \leq \gamma(V)$, $\mu \leq \gamma(U)$ and $\gamma(V)$, $\gamma(U)$ are fuzzy semi-closed set. This

implies that $FScl(\lambda) \leq FScl(\gamma(V)) = \gamma(v) \leq \gamma(\mu)$ and $FScl(\mu) \leq \gamma(\lambda)$. Therefore $\lambda \wedge FScl(\mu) = 0 = FScl(\lambda) \wedge \mu$. Hence λ and μ are fuzzy semi-separate sets.

Proposition 4.6. *Any two fuzzy semi-separated sets are disjoint sets.*

Proof : Let λ and μ be fuzzy semi-separated. Then $\lambda \wedge FScl(\mu) = 0 = FScl(\lambda) \wedge \mu$. Now $\lambda \wedge \mu \leq \lambda \wedge FScl(\mu) = 0$. Therefore $\lambda \wedge \mu = 0$. Hence λ and μ are disjoint. The converse of the above preposition need not be true which is shown by the following example.

Example 4.7. Let $X = \{a, b\}$ with $\rho = \{0, \lambda, \mu, \gamma, 1\}$ and $\gamma(\rho) = \{1, \gamma(A), \gamma(B),$
$\gamma(C), 0\}$ where $\lambda = \langle(a, 0.3, 0, 0.5), (b, 0, 0.3, 1)\rangle, \mu = \langle(a, 0, 0.4, 1), (b, 0.6, 0, 0.8)\rangle,$
$\gamma = \langle(a, 0.3, 0.4, 0.5), (b, 0.6, 0.3, 0.8)\rangle, \gamma(\lambda) = \langle(a, 0.5, 1, 0.3), (b, 1, 0.7, 0)\rangle,$
$\gamma(\mu) = \langle(a, 1, 0.6, 0), (b, 0.8, 1, 0.6)\rangle$, and $\gamma(c) = \langle(a, 0.5, 0.6, 0.3), (b, 0.8, 0.7, 0.6)\rangle$, Then $\lambda \wedge \mu = 0$. Then λ and μ are disjoint fuzzy sets. Now $FScl(\lambda) = \gamma(c)$ and $FScl(\mu) = \gamma(c)$. Thus $FScl(\lambda) \wedge \mu = \gamma(c) \wedge \mu = \mu \neq 0$ and $\lambda \wedge FScl(\mu) = A \wedge \gamma(c) = \lambda \neq 0$. Therefore λ and μ are not fuzzy semi-separated sets.

Proposition 4.8. *If fuzzy set λ in a fuzzy topological space (C, ρ) is fuzzy semi-connected if and only if λ is not the union if any two fuzzy semi-separated sets.*

Proof : Let (C, ρ) be a fuzzy semi-connected space and let $\lambda = \alpha \vee \beta$ where α and β are fuzzy semi-separated sets. Then by definition 4.1 $FScl(\alpha) \wedge \beta = \alpha \wedge FScl(\beta) = 0$. since $\alpha \leq FScl(\alpha), \alpha \wedge \beta \leq FScl(\alpha) \wedge \beta = 0$. Thus $\alpha \wedge \beta = 0$. Also $FScl(\alpha) \leq c(\beta) = \alpha$ and $FScl(\beta) \leq c(\alpha) = \beta$. Hence $\alpha = FScl(\alpha)$ and $\beta = FScl(\beta)$. Therefore α and β are fuzzy semi-closed sets. Hence $\alpha = c(\beta)$ and $\beta = c(\alpha)$ are disjoint fuzzy semi-closed sets. That is, λ is not fuzzy semi-connected, which is a contradiction to λ is a fuzzy semi-connected space. Hence λ is not the union of any two fuzzy semi-separated sets.
Conversely. assume that λ is not the union of any two fuzzy semi-separated sets. Let (C, ρ) be a fuzzy semi-disconnected space. Then $\lambda = \alpha \vee \beta$ where α and $\beta$are non-empty disjoint fuzzy semi-open sets in (C, ρ). Since $\alpha \leq c(\beta)$ and $\beta \leq c(\alpha)$, $FScl(\alpha) \wedge \beta \leq c(\beta) \wedge \beta = 0$

and $\alpha \wedge FScl(\beta) \leq \alpha \wedge c(\alpha) = 0$. Thus α and β are fuzzy semi-separated sets. This is a contradiction to our assumption. Therefore λ is fuzzy semi-connected.

Proposition 4.9. *A fuzzy topological space (C, ρ) is a fuzzy semi-connected space if and only if 1 is not the union of any two fuzzy semi-separated sets.*

Proof : Let (C, ρ) be a fuzzy semi-connected space and let $1 = \lambda \vee \mu$ where λ and μ are fuzzy semi-separated sets. Then by Definition 4.1, $FScl(\lambda) \wedge \mu = \lambda \wedge FScl(\mu) = 0$. Since $\lambda \leq FScl(\lambda), \lambda \wedge \mu \leq FScl(\lambda) \wedge \mu = 0$. Thus $\lambda \wedge \mu = 0$. Also $FScl(\lambda) \leq c(\mu) = \lambda$ and $FScl(\mu) \leq c(\lambda) = \mu$. Hence $\lambda = FScl(\lambda)$ and $\mu = FScl(\mu)$. Therefore λ and μ are fuzzy semi-closed sets. Hence $\lambda = c(\mu)$ and $\mu = c(\lambda)$ are disjoint fuzzy semi-open sets. That is, 1 is not fuzzy semi-connected, Which is a contradiction to (C, ρ) is a fuzzy semi-connected space. Hence 1 is not the union of any two fuzzy semi-separated sets.
Conversely, assume that 1 is not the union of any two fuzzy semi-separated sets. Let $(c.\rho)$ be a fuzzy semi-disconnected space. Then $1 = \lambda \vee \mu$ where λ and μ are non-empty disjoint fuzzy semi-open sets in (C, ρ). Since $\lambda \leq c(\mu)$ and $\mu \leq c(\lambda)$, $FScl(\lambda) \wedge \mu \leq c(\mu) \wedge \mu = 0$ and $\lambda \wedge FScl(\mu) \leq \lambda \wedge c(\lambda) = 0$. Thus λ and μ are fuzzy semi-separated sets. This is a contradiction to our assumption. Therefore $(c.\rho)$ is fuzzy semi-connected.

Proposition 4.10. *A fuzzy topological space (C, ρ) is fuzzy semi-connected if and only if the union of any two fuzzy semi-separated sets is not equal to 1.*

Proof : Let (C, ρ) be fuzzy semi-connected space and let $1 = \lambda \wedge \mu$ where λ and μ are fuzzy semi-separated sets. Then by Definition 4.1, $FScl(\lambda) \wedge \mu = \lambda \wedge FScl(\mu) = 0$. Since $\lambda \leq FScl(\lambda), \lambda \wedge \mu \leq FScl(\lambda) \wedge \mu = 0$. Thus $\lambda \wedge \mu = 0$. Also $FScl(\lambda) \leq c(\mu) = \lambda$ and $FScl(\mu) \leq c(\lambda) = \mu$. Hence $\lambda = FScl(\lambda)$ and $\mu = FScl(\mu)$. Therefore λ and μ are fuzzy semi-closed sets. Hence $\lambda = c(\mu)$ and $\mu = c(\lambda)$ are disjoint fuzzy semi-open sets. That is, 1 is not fuzzy semi-connected. Which is a contradiction to (C, ρ) is a fuzzy semi-connected space. Hence the union of any two fuzzy semi-separated sets is not equal to 1.
Conversely, assume that union of any two fuzzy semi-separated sets is not equal to 1. Let (C, ρ) be a fuzzy semi-disconnected space. Then $1 = \lambda \vee \mu$ where λ and μ are non-empty

disjoint fuzzy semi-open sets in (C, ρ). Since $\lambda \leq c(\mu)$ and $\mu \leq c(\lambda)$, $FScl(\lambda) \wedge \mu \leq c(\mu) \wedge \mu = 0$ and $\lambda \wedge FScl(\mu) \leq \lambda \wedge c(\lambda) = 0$. Thus λ and μ are fuzzy semi-separated sets. This is a contradiction to our assumption. Therefore (C, ρ) is fuzzy semi-connected.

Proposition 4.11. *If $\lambda \leq H \vee K$ where λ is a fuzzy semi-connected set and H, K are fuzzy semi-separated sets, then either $\lambda \leq H$ or $\lambda \leq K$.*

Proof : Suppose $\lambda \not\leq H$ and $\lambda \not\leq K$. Let $\lambda_1 = H \wedge \lambda$ and $\lambda_2 = K \wedge \lambda$. Then λ_1 and λ_2 are non-empty fuzzy sets and $\lambda_1 \vee \lambda_2 = (H \wedge \lambda) \vee (K \wedge \lambda) = (H \vee K) \wedge \lambda = \lambda$, because $\lambda \leq H \vee K$. Since $\lambda_1 \leq H, \lambda_2 \leq K$, H, K are fuzzy semi-separated sets, $FScl(\lambda_1) \wedge \lambda_2 \leq FScl(H) \wedge K = 0$ and $\lambda_1 \wedge FScl(\lambda_2) \leq H \wedge FScl(K) = 0$. Therefore λ_1, λ_2 are fuzzy semi-separated sets such that $\lambda = \lambda_1 \vee \lambda_2$. Hence by Proposition 4.8, λ is fuzzy semi-disconnected. This is a contradiction to λ is fuzzy semi-connected. Thus either $\lambda \leq H$ or $\lambda \leq K$.

Proposition 4.12. *If λ is a fuzzy semi-connected set, then $FScl(\lambda)$ is also a fuzzy semi-connected set.*

Proof : If λ be a fuzzy semi-connected set and $FScl(\lambda)$ be fuzzy semi-disconnected. Then by proposition 4.8, there exist fuzzy semi-separated sets H and K such that $FScl(\lambda) = H \vee K$. Since λ is fuzzy semi-connected and $\lambda \leq FScl(\lambda) = H \vee K$, by Proposition 4.11, either $\lambda \leq H$ or $\lambda \leq K$. Since $\lambda \leq H$ and by Lemma 2.9(iii), $FScl(\lambda) \leq FScl(H)$. Again since H and K are fuzzy semi-separated sets, $H \neq 0, K \neq 0$ and $FScl(\lambda) \wedge K \leq FScl(H) \wedge K = 0$. Thus $K \leq c(FScl(\lambda))$. Also $K \leq H \cup K = FScl(\lambda)$. Therefore $K \leq c(FScl(\lambda)) \wedge FScl(\lambda) = 0$ which is a contradiction to $K \neq 0$. Similarly if $\lambda \leq K$, We get a contradiction to $H \neq 0$. Therefore $FScl(\lambda)$ is fuzzy semi-connected.

Proposition 4.13. *Let β be a fuzzy semi-connected in (C, ρ). If $\beta \leq \delta \leq FScl(\beta)$, then δ is fuzzy semi-connected.*

Proof : Suppose δ is fuzzy semi-disconnected. Then there exist non-empty fuzzy semi-separated set λ and μ such that $\delta = \lambda \vee \mu$. Since $\beta \leq \delta = \lambda \vee \mu$ and by Proposition

4.11, $\beta \leq \lambda$ or $\beta \leq \mu$. Let $\beta \leq \lambda$. Then by Lemma 2.9(iii), $FScl(\beta) \leq FScl(\lambda)$. Now $FScl(\beta) \wedge \mu \leq FScl(\lambda) \wedge \mu = 0$. Thus $FScl(\beta) \wedge \mu = 0$. Also $\lambda \vee \mu = \delta \leq FScl(\beta)$, $\mu \leq \delta \leq FScl(\beta)$. Therefore $FScl(\beta) \wedge \mu = \mu$. Hence $\mu = 0$. This is a contradiction to $\mu \neq 0$. Hence δ is fuzzy semi-connected.

Proposition 4.14. *If every two fuzzy points of a fuzzy set β in (C, ρ) are contained in some fuzzy semi-connected set of β, then β is a fuzzy semi-connected set in (C, ρ).*

Proof : Suppose β is fuzzy semi-disconnected. Then there exist non-empty separated sets λ and μ of (C, ρ) such that $\beta = \lambda \cup \mu$. Since λ and μ are non-empty there exists a fuzzy points $X(\alpha, \beta, \lambda) \in \lambda$ and $Y(\gamma, \delta, \eta) \in \mu$. By hypothesis, $X(\alpha, \beta, \lambda)$ and $Y(\gamma, \delta, \eta)$ must be contained in some fuzzy semi-connected set δ of β. Since $\delta \leq \lambda \vee \mu$ and by Proposition 4.11, $\delta \leq \lambda$ and $\delta \leq \mu$. It follows that either $X(\alpha, \beta, \lambda), Y(\gamma, \delta, \eta)$ are both in λ or both in μ. Let $X(\alpha, \beta, \lambda)$ and $Y(\gamma, \delta, \eta) \in \lambda$. Then $Y(\gamma, \delta, \eta) \in \mu$, $\lambda \wedge \mu \neq 0$. This is a contradiction to $\lambda \wedge \mu = 0$. Hence β is fuzzy semi-connected.

Proposition 4.15. *If λ and μ are the fuzzy semi-connected space in (C, ρ) such that $\lambda \wedge \mu \neq 0$, then $\lambda \wedge \mu$ is a fuzzy semi-connected space in (C, ρ)*

Proof : Let λ and μ be fuzzy semi-disconnected. Then by Proposition 4.9, there exist two fuzzy semi-separated sets H, K such that $\lambda \vee \mu = H \vee K$, Since H and K are fuzzy semi-separated, H and K are non-empty fuzzy sets and $H \wedge K \leq FScl(H) \wedge K = 0$. Since $\lambda \leq \lambda \vee \mu = H \vee K$, $\mu \leq \lambda \vee \mu = \lambda \vee \mu$ and λ, μ are fuzzy semi-connected, by Proposition 4.11, $\lambda \leq H$ or $\lambda \leq K$ and $\mu \leq H$ or $\mu \leq K$.

(1) If $\lambda \leq H$ and $\mu \leq H$, then $\lambda \wedge \mu \leq H$. Thus. $\lambda \vee \mu = H$. Since H and K are disjoint $K = 0$. This is a contradiction to $K \neq 0$. Similarly, if $\lambda \leq K$ and $\mu \leq K$. We get a contradiction

(2) If $\lambda \leq H$ and $\mu \leq K$, then $\lambda \wedge \mu \leq H \wedge K = 0$. Therefore $\lambda \wedge \mu = 0$. This is a contradiction to $\lambda \wedge \mu \neq 0$. Similarly, We get a contradiction if $\lambda \leq K$ and $\mu \leq H$. Therefore $\lambda \vee \mu$ is fuzzy semi-connected in (C, ρ).

Proposition 4.16. (i) *Let $f : X \to Y$ be fuzzy semi-irresolute and (C, ρ) be fuzzy semi-connected. Then (Y, S) is fuzzy semi-connected.*
(ii) *Let $f : X \to Y$ be fuzzy semi-irresolute surjection and (C, ρ) be fuzzy semi-connected. Then (Y, S) is fuzzy semi-connected.*

Proof : (i) Suppose (Y, S) is semi-disconnected. Then $Y = \lambda \vee \mu$ where λ and μ are disjoint non-empty semi-open sets in Y. Since f is fuzzy semi-irresolute surjection, $1 = f^{-1}(\lambda) \vee f^{-1}(\mu)$ where $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are disjoint non-empty semi-open sets in (C, ρ). This is a contradiction to (C, ρ) is fuzzy semi-connected. Hence (Y, S) is fuzzy semi-connected.
(ii) Suppose (Y, S) is fuzzy semi-disconnected. Then $Y = \lambda \vee \mu$ where λ and μ are disjoint non-empty fuzzy semi-open sets in Y, since f is fuzzy semi-irresolute surjection, $1 = f^{-1}(\lambda) \vee f^{-1}(\mu)$ where $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are disjoint non-empty fuzzy semi-open sets in X. This is a contradiction to (C, ρ) is fuzzy semi-connected. Hence (Y, S) is fuzzy semi-connected.

Proposition 4.17. *Let $f : X \to Y$ be fuzzy semi-open, fuzzy semi-closed and injection and (Y, S) be fuzzy semi-connected. Then (C, ρ) be fuzzy semi-connected.*

Proof : Let f be fuzzy semi-open and fuzzy semi-closed. Then $f(\lambda)$ is both fuzzy semi-open and fuzzy semi-closed. Since (Y, S) is fuzzy semi-connected by Proposition 3.8, $f(\lambda) = 0$ or $f(\lambda) = 1$. Since f is injection, $\lambda = 0$ or $\lambda = 1$. Hence by Proposition 3.8, (C, ρ) is fuzzy semi-connected.

5. Conclusion

In this paper, We have examined fuzzy semi-connected spaces and fuzzy semi-separated sets. Also, we examined their properties and interrelations with examples.

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Fuzzy Strongly Nodec Spaces

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Abstract. In this paper is an attempt to study and introduce the notion of fuzzy strongly boundary set and fuzzy strongly nodec spaces in fuzzy topological spaces. Several examples are given to illustrate the concepts introduced in this paper. By means of fuzzy strongly boundary set, the notion of fuzzy strongly nodec spaces defined and several characterizations of fuzzy strongly boundary set and fuzzy strongly nodec spaces are obtained.

Keywords: Fuzzy strongly boundary set, fuzzy strongly nowhere dense set, fuzzy strongly first and second category, fuzzy strongly residual set and fuzzy strongly nodec spaces.

1. Introduction

The concepts of Fuzzy sets and fuzzy set operations were first introduced by L.A.Zadeh in his classical paper [11] in the year 1965. Thereafter the paper of C.L.Chang [4] in 1968 paved the way for the subsequent tremendous growth of the numerous Fuzzy topological concepts. Since then much alteration has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed in X.Tang [6] used a slightly changed version of Chang's fuzzy topological spaces to model spatial objects for GIS data bases and Structured Query Language (SQL) for GIS.

Through Pu and Liu [5] defined the notion of fuzzy boundary in fuzzy topological spaces in 1980. Ahmed and Athar [1] defined the concepts of fuzzy semi boundary and characterized fuzzy semi continuous functions. In terms of fuzzy semi boundary.

In this paper, we present several properties of fuzzy strongly boundary and fuzzy strongly nodec spaces which have been supported by examples. In section 4, the concepts of fuzzy strongly nodec spaces and several characterization of inter relationships between fuzzy strongly boundary set and fuzzy strongly nodec spaces are investigated.

2. Preliminaries

First, we briefly recall certain definitions and results are described.
A fuzzy set λ in a set X is a function from X to [0,1], that is, λ: $X \to [0,1]$.

Definition 2.1. [4] Let λ and μ be any two fuzzy sets in (X,T). Then we define $\lambda \vee \mu : X \to$ [0,1] as follows: $(\lambda \vee \mu)(x) = Max\{\lambda(x), \mu(x)\}$. Also we define $\lambda \wedge \mu : X \to [0, 1]$ as follows: $(\lambda \wedge \mu)(x) = Min\{\lambda(x), \mu(x)\}$.

Definition 2.2. [4] Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define $\text{int}(\lambda) = \vee \{\mu/\mu \leq \lambda, \mu \in T\}$ and $\text{cl}(\lambda) = \wedge \{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.
For any fuzzy set in a fuzzy topological space (X,T), it is easy to see that
1 – cl(λ) =int (1 – λ) and 1 – int(λ) =cl (1 – λ).

Lemma 2.3. *[2] For a fuzzy set λ of a fuzzy topological space (X,T),*
(i)1 – int (λ) =cl (1 – λ),
(ii) 1 – cl(λ) =int (1 – λ).

Definition 2.4. [8] A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, cl(λ)=1 .

Definition 2.5. [7] A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu <$ cl(λ). That is, int cl (λ) = 0.

Definition 2.6. [3] A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy G_δ-set in (X,T) if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$ where 1 - $\lambda_i \in$ T for i $\in$ I.

Definition 2.7. [3] A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy F $_\sigma$-set in (X,T) if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$ where 1 - $\lambda_i \in$ T for i $\in$ I.

Definition 2.8. [7] A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X, T) is said to be of second category.

Definition 2.9. [8] Let λ be a fuzzy first category set in a fuzzy topological space in (X,T). Then 1 - λ is called a fuzzy residual set in (X,T).

Definition 2.10. [8] A fuzzy topological space (X,T) is called fuzzy first category if the fuzzy set 1_X is a fuzzy first category space in (X,T). That is $1_x = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X T). Otherwise (X,T) will be called a fuzzy second category.

Lemma 2.11. *[5] For a family of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X,T), $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case A is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$*
Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$

Definition 2.12. [10] Let (X,T) be a fuzzy topological spaces. A fuzzy set λ defined on X is called a fuzzy strongly nowhere dense set, if $\lambda \wedge (1-\lambda)$ is a fuzzy nowhere dense set in (X,T). That is, λ is a fuzzy nowhere dense set in (X,T), if $intcl[\lambda \wedge (1-\lambda)] = 0$ in (X,T).

Definition 2.13. [10] A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy strongly first category if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy strongly nowhere dense sets in (X,T). Any other fuzzy set in (X, T) is said to be of fuzzy strongly second category.

Definition 2.14. [10] Let λ be a fuzzy strongly first category set in(X,T). Then 1 - λ is called a fuzzy residual set in (X,T).

3. **Fuzzy Strongly Boundary Set**

Definition 3.1. Let λ be a fuzzy set in a fuzzy topological space (X,T). Then the fuzzy strongly boundary of λ is defined as $\text{Bd}(\lambda) = cl(\lambda) \wedge cl(1-\lambda)$, $\text{Bd}(\lambda)$ is a fuzzy closed set in (X,T) .

Example 3.2. Let X=$\{a, b, c\}$ and λ, μ, γ be the fuzzy sets defined on X as follows:
λ: X $\to$ [0,1] is defined by λ(a)=0.7, λ(b)=0.4, λ(c)=0.6
μ: X $\to$ [0,1] is defined by μ(a)=0.6, μ(b)=0.6, μ(c)=0.5
γ: X $\to$ [0,1] is defined by γ(a)=0.5, γ(b)=0.6, γ(c)=0.7
Then T=$\{0, \lambda, \mu, \gamma, \lambda\vee\mu, \mu\vee\gamma, \lambda\vee\gamma, \lambda\wedge\mu, \mu\wedge\gamma, \lambda\wedge\gamma, \mu\vee(\lambda\wedge\gamma), \lambda\wedge(\mu\vee\gamma), \mu\wedge(\lambda\vee\gamma), \gamma\wedge(\lambda\vee\mu), 1\}$ is a fuzzy topology on I. Now $1-\lambda, 1-\mu, 1-\lambda\vee\mu, 1-\lambda\wedge\mu$ and 1 are fuzzy closed sets in (X,T).On computation, we see that $cl(\lambda) \wedge cl(1-\lambda) = 1-\lambda\vee\mu; cl(\mu) \wedge cl(1-\mu) = 1-\lambda\vee\mu; cl(\lambda\vee\mu)\wedge cl(1-\lambda\vee\mu) = 1-\lambda\vee\mu$. $Bd(\lambda) = 1-\lambda\vee\mu \implies intcl[(\lambda\vee\mu)\wedge(1-\lambda\vee\mu)] = intcl(1-\lambda\vee\mu) = int(1-\lambda\vee\mu) = 0$ and hence $\text{Bd}(\lambda)$ is a fuzzy strongly nowhere dense set in (X,T). Also, $\text{Bd}(\mu)$ and $\text{Bd}(\lambda\vee\mu)$ is a fuzzy strongly nowhere dense sets in (X,T). Hence $\text{Bd}(\lambda)$, $\text{Bd}(\mu)$ and $\text{Bd}(\lambda\vee\mu)$ is a fuzzy strongly boundary sets in (X,T).

Proposition 3.3. *If λ is a fuzzy strongly nowhere dense set in (X,T), then Bd(λ) is a fuzzy strongly nowhere dense set in (X,T).*

Proof. Let λ be a fuzzy strongly nowhere dense set in (X,T). Then $intcl[\lambda \wedge (1-\lambda)] = 0$ in (X,T). Since $\lambda\wedge(1-\lambda) \leq cl(\lambda)\wedge cl(1-\lambda)$ in (X,T), $intcl[(\lambda)\wedge(1-\lambda)] \leq intcl[cl(\lambda)\wedge cl(1-\lambda)]$

and hence $0 \leq intcl[cl(\lambda) \wedge cl(1-\lambda)]$. That is, $intcl[cl(\lambda) \wedge cl(1-\lambda)] = 0$. This implies that $intcl[Bd(\lambda)] = 0$. Hence Bd(λ) is a fuzzy strongly nowhere dense set in (X,T). □

Proposition 3.4. *If Bd(λ) is a fuzzy strongly nowhere dense set in (X,T) then Bd(int λ) is a fuzzy strongly nowhere dense set in (X,T).*

Proof. Let Bd(λ) is a fuzzy strongly nowhere dense set in (X,T). Then $intcl[Bd(\lambda)] = 0$. This implies that $intcl[cl(\lambda) \wedge cl(1-\lambda)] = 0$. Now, $Bd(int\lambda) = cl(int\lambda) \wedge cl(1-int\lambda)$. This implies that $Bd(int\lambda) = clint\lambda \wedge cl(cl(1-\lambda)) = clint\lambda \wedge cl(1-\lambda)$ Now $Bd(int\lambda) \subset cl(\lambda) \wedge cl(1-\lambda) \subset Bd(\lambda)$. Thus $Bd(int\lambda) \subset Bd(\lambda)$. By Proposition 3.3, Bd(λ) is afuzzy strongly nowhere dense set in (X,T). Also Bd(int λ) is a subset of Bd(λ). Therefore Bd($int\lambda$) is also a fuzzy strongly nowhere dense set in (X,T). □

Proposition 3.5. *If Bd(λ) is a fuzzy strongly nowhere dense set in (X,T), then Bd(cl λ) is a fuzzy strongly nowhere dense set in (X,T).*

Proof. Let Bd(λ) is a fuzzy strongly nowhere dense set in (X,T). Then $intcl[Bd(\lambda)] = 0$. This implies that $intcl[cl(\lambda) \wedge cl(1-\lambda)] = 0$. Now, $Bd(cl\lambda) = cl(cl\lambda) \wedge cl(1-cl\lambda)$. This implies that $Bd(cl\lambda) = cl(\lambda) \wedge cl(int(1-\lambda)) = cl(\lambda) \wedge clint(1-\lambda)$. Now $Bd(cl\lambda) \subset cl(\lambda) \wedge cl(1-\lambda) \subset Bd(\lambda)$. Thus $Bd(cl\lambda) \subset Bd(\lambda)$. By Proposition 3.3, Bd(λ) is a fuzzy strongly nowhere dense set in (X,T). Also Bd(cl λ) is a subset of Bd(λ). Therefore Bd($cl\lambda$) is also a fuzzy strongly nowhere dense set in (X,T). □

Proposition 3.6. *If Bd(λ) and Bd(μ) are fuzzy strongly nowhere dense set in a fuzzy topological space (X,T), then Bd(int $\lambda \wedge \mu$) is a fuzzy strongly nowhere dense set in (X,T).*

Proof. Let Bd(λ) and Bd(μ) are fuzzy strongly nowhere dense set in (X,T). Then $intcl[Bd(\lambda)] = 0$ *and* $intcl[Bd(\mu)] = 0$. This implies that $intcl[cl(\lambda) \wedge cl(1-\lambda)] = 0$ *and* $intcl[cl(\mu) \wedge cl(1-\mu)] = 0$in (X,T). Now, $intcl[Bd(\lambda \wedge \mu)] = intcl[cl(\lambda \wedge \mu) \wedge cl(1-\lambda \wedge \mu)] = 0$. Then By Proposition 3.3, λ is a fuzzy strongly nowhere dense set in (X,T), then Bd(λ) is a fuzzy strongly nowhere dense set in (X,T) which implies that $\lambda \wedge \mu$ is a subset of λ and μ, λ and μ is a fuzzy strongly nowhere dense set in (X,T) and Bd($\lambda \wedge \mu$) is a fuzzy strongly nowhere dense sets in (X,T). □

Example 3.7. We follow the Ex.3.2, Bd(λ) and Bd(μ) is a fuzzy strongly nowhere dense set in (X,T) which implies that Bd($\lambda \wedge \mu$) $= intcl[(\lambda \wedge \mu) \wedge (1-\lambda \wedge \mu)] = intcl(1-\mu) = int(1-\mu) = 0$. And hence Bd($\lambda \wedge \mu$) is a fuzzy strongly nowhere dense sets in (X,T).

4. **Fuzzy Strongly Nodec Space**

Definition 4.1. A fuzzy topological space (X,T) is called a fuzzy strongly nodec space if every fuzzy strongly nowhere dense set in (X,T) is a fuzzy closed set in (X,T).

Example 4.2. Let X=$\{a, b, c\}$. Let I=[0,1]. Then the fuzzy sets λ, μ, γ and δ are defined on X as follows:

λ: X $\to$ [0,1] is defined by λ(a)=0.5, λ(b)=0.6, λ(c)=0.7
μ: X $\to$ [0,1] is defined by μ(a)=0.4, μ(b)=0.8, μ(c)=0.3
γ: X $\to$ [0,1] is defined by γ(a)=0.5, γ(b)=0.6, γ(c)=0.6
δ: X $\to$ [0,1] is defined by δ(a)=0.4, δ(b)=0.6, δ(c)=0.3 Then, T = $\{ 0, \lambda, \mu, \gamma, 1 \}$ is a fuzzy topology on X. On computation we see that cl(λ) = 1; cl(μ) = 1, cl(γ)=1, cl($\lambda \vee \mu$) = 1, cl($\mu \vee \gamma$) = 1, cl($\lambda \wedge \mu$) = 1, int($1 - \lambda$)=0, int($1 - \mu$)=0, int($1 - \gamma$)=0, int($1 - \lambda \vee \mu$)=0, int($1 - \mu \vee \gamma$)=0, int($1 - \lambda \vee \mu$)=0 in (X,T).

Now, int[cl(δ)] = int(1-$\lambda \wedge \mu$)=0 and hence δ is a fuzzy nowhere dense set in (X,T). Also int[$\delta \wedge (1 - \delta)$] = $int(1 - \lambda) = 0$ and hence δ is a fuzzy strongly nowhere dense set in (X,T).

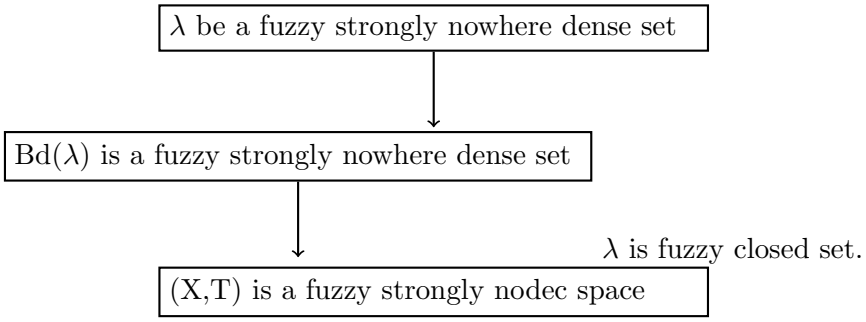
Now, int[cl(λ)] = int(1) $\neq$ 0 and hence λ is not a fuzzy nowhere dense set in (X,T). Also int cl[$\lambda \wedge (1 - \lambda)$] = $int[cl(1 - \lambda)] = int(1 - \lambda) = 0$ and hence λ is a fuzzy strongly nowhere dense set in (X,T). Which implies that fuzzy strongly nowhere dense sets is a fuzzy closed set in (X,T). Hence (X,T) is fuzzy strongly nodec space.

Proposition 4.3. *If λ is a fuzzy strongly nowhere dense set in a fuzzy strongly nodec space then Bd(λ) is a fuzzy strongly nowhere dense set in (X,T).*

Proof. Let λ be a fuzzy strongly nowhere dense set in (X,T). Then $intcl[\lambda \wedge (1 - \lambda)] = 0$ in (X,T). Since (X,T) is a fuzzy strongly nodec space. By hypothesis, λ is a fuzzy closed set in (X,T). Now Bd(λ) = cl(λ) $\wedge$ ($1 - \lambda$) = $\lambda \wedge 1 - \lambda$. Therefore Bd($\lambda$) $\subset \lambda, \lambda$ is a fuzzy strongly nowhere dense set in (X,T) and hence Bd(λ) is a fuzzy strongly nowhere dense set in (X,T). □

Proposition 4.4. *Let (X,T) be a fuzzy topological space. If Bd(λ) is a fuzzy strongly nowhere dense set λ is a fuzzy closed set in (X,T), then (X,T) is a fuzzy strongly nodec space.*

Proof


λ be a fuzzy strongly nowhere dense set
Bd(λ) is a fuzzy strongly nowhere dense set
λ is fuzzy closed set.
(X,T) is a fuzzy strongly nodec space

Proposition 4.5. *Let (X,T) be a fuzzy topological space. Then the following are equivalent*
(i) (X,T) is a fuzzy strongly nodec space
(ii) For every fuzzy strongly first category set λ is a fuzzy $F_\sigma-$ set in (X,T).
(iii) For every fuzzy strongly residual category set μ is a fuzzy $G_\delta-$ set in (X,T).

Proof. (1) $\Rightarrow$ (2). Let λ be a fuzzy first category set in (X,T). Then $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy strongly nowhere dense sets in (X,T). since (X,T) is a fuzzy strongly nodec space, every fuzzy strongly nowhere dense sets in (X,T) is a fuzzy closed sets in (X,T). This implies that $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $(1-\lambda_i) \in T$ and hence λ is a fuzzy $F_\sigma-$ set in (X,T).

(2) $\Rightarrow$ (3). Let μ be a fuzzy residual category set in (X,T). Then $1-\mu$ is a fuzzy strongly first category set in (X,T). Now $1-\mu = \bigvee_{i=1}^{\infty}(1-\mu_i)$,$(1-\mu_i)$'s are fuzzy strongly nowhere dense sets in (X,T) and by hypothesis $1-\mu_i$ is a fuzzy closed sets in (X,T). Then $1-\mu = 1-\bigwedge_{i=1}^{\infty}(\mu_i) \implies \mu = \bigwedge_{i=1}^{\infty}\mu_i,\ \mu_i \in T$ and hence μ is a fuzzy $G_\delta-$ set in (X,T).

(3) $\Rightarrow$ (1).Let λ be a fuzzy first category set in (X,T). Then $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy strongly nowhere dense sets in (X,T). Now λ is fuzzy strongly first category set in (X,T) implies that $1-\lambda$ is a fuzzy strongly residual category set in (X,T). By hypothesis $1-\lambda$ is a fuzzy closed sets in (X,T). Then (λ_i)'s are fuzzy strongly nowhere dense sets and fuzzy closed sets in (X,T). Hence (X,T) is a fuzzy strongly nodec space. □

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Applications of Fuzzy Baire Spaces

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Abstract. The fuzzy sets, in which the elements of the universe have their membership and non-membership degrees in [0,1] is of Zadeh's fuzzy set. In this paper fuzzy sets are used as tools for assessment and decision making. This is useful in cases where one is not sure about the suitability of the linguistic characterizations assigned to each element of the universal set. Applications illustrating our results are also presented.

Keywords: Fuzzy nowhere desne set, fuzzy first category set, fuzzy Baire space, Fuzziness and Vagueness, Fuzzy interior and closure, Fuzzy Decision making, Fuzzy information systems

1. Introduction

Assessment is a very important process connected to all human and machine activities, because it helps to improve the performance by avoiding mistakes or weak behaviours of the past. In many cases the assessment is performed using qualitative grades instead of numerical scores. This happens either because the existing data are not exact, or for reasons of elasticity. Obviously in such cases the mean performance of a group cannot be assessed calculating the mean value of the individual scores of its members.

Decision making the other hand, which is a fundamental activity of human cognition, can be defined as the process of choosing a solution among two or more alternatives, with the help of suitable criteria, among to obtained the the best possible outcomes for a given problem. The recent technological process and the rapid changes happening to human society increased the complexity of the decision making problems. Thus from the 1950's a systematic approach has been started for decision making known as Statistical Decision Theory, which is based on a synthesis of principles of several scientific fields, including Probability Theory, Statistics, Economics, Psychology.

Frequently in everyday life situations, however the decision making process takes place under conditions of uncertainty, which makes the applications of the traditional Decision Making

methods for the solution of the corresponding problems impossible. The application of fuzzy sets to assessment process is developed in section 4 also their application to decision making. Finally, includes the general conclusions and a short discussion for further research.

2. **Preliminaries**

Definition 2.1. [4] Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define $\text{int}(\lambda) = \vee \{\mu/\mu \leq \lambda, \mu \in T\}$ and $\text{cl}(\lambda) = \wedge \{\mu/\lambda \leq \mu, 1-\mu \in T\}$.
For any fuzzy set in a fuzzy topological space (X,T), it is easy to see that
1 – cl(λ) =int (1 – λ) and 1 – int(λ) =cl (1 – λ).

Definition 2.2. [4] A fuzzy set λ in a fuzzy topological space X is called fuzzy open if $\lambda \leq clint(\lambda)$ and fuzzy closed if $intcl(\lambda) \leq \lambda$.

Definition 2.3. [4] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy G_δ -set in (X, T) if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in$ T.

Definition 2.4. [4] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ -set in (X, T) if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where 1 - $\lambda_i \in$ T.

Definition 2.5. [4] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.

Definition 2.6. [3] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.7. [3] Let λ be a fuzzy first category set in a fuzzy topological space (X, T). Then 1 - λ is called a fuzzy residual set in (X, T).

Definition 2.8. [4] Let(X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called fuzzy σ-first category if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)]$ where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be fuzzy σ-second category in (X,T) .

Definition 2.9. [4] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy σ-nowhere dense set, if λ is a fuzzy F_σ-set in (X, T) such that $\text{int}(\lambda) = 0$.

Definition 2.10. [3] Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if $int\,[\bigvee_{i=1}^{\infty}(\lambda_i)] = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T).

Definition 2.11. [4] Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy σ-Baire space if $int\,[\bigvee_{i=1}^{\infty}(\lambda_i)] = 0$ where (λ_i)'s are fuzzy σ-nowhere dense sets in (X,T).

3. **Application**

Fuzzy Baire spaces are important in areas where uncertainty or vagueness is inherent, such as

∗ Decision theory

∗ Fuzzy control systems

∗ Approximate reasoning

∗ Fuzzy neural networks

∗ Optimization problems with imprecise data.

In such systems, preserving properties like "non-meagerness" (i.e., avoiding the space being a countable union of fuzzy nowhere-dense sets) crucial to ensure the existence of meaningful solutions or stable states.

4. **Relevance of fuzzy Baire spaces in Decision Theory**

Decision Theory involves choosing the best option under uncertainty. Traditional models assume Crisp, precise, information: however, real-world decisions often involve vague, imprecise, or incomplete information, which is better handled by fuzzy set theory.

A fuzzy Baire space ensures that the space of possible decision is "rich" or "non - meager", meaning viable decision aren't lost due to overly narrow or restrictive criteria. This concept is crucial in ensuring preferences or data don't eliminate all acceptable outcomes.

Example 4.1. A company wants to employ a new person among the seven candidates A,B,C,D,E,F and G. The ideal qualifications for the new employee are to have satisfactory previous experience and other categories [eg, to strength of subjective, to be driving license and to be young]. Assume that A, B, G are are the candidate with satisfactory previous experience B,C,D,E,F are the strength of subjective, D, G. are the holders of a driving license and D is the only young candidate. Find the best decision for the company.

Solution: Let T={A,B,C,D,E,F,G} and let X=$\{\lambda, \mu, \gamma, \delta\}$ be the set of parameters λ = well experienced, μ = subject strength γ= holder of a driving license and δ =young candidate.

X: [0,1] is defined as

λ={ A,B,G}

μ={B,C,D,E,F}

γ={D,G}

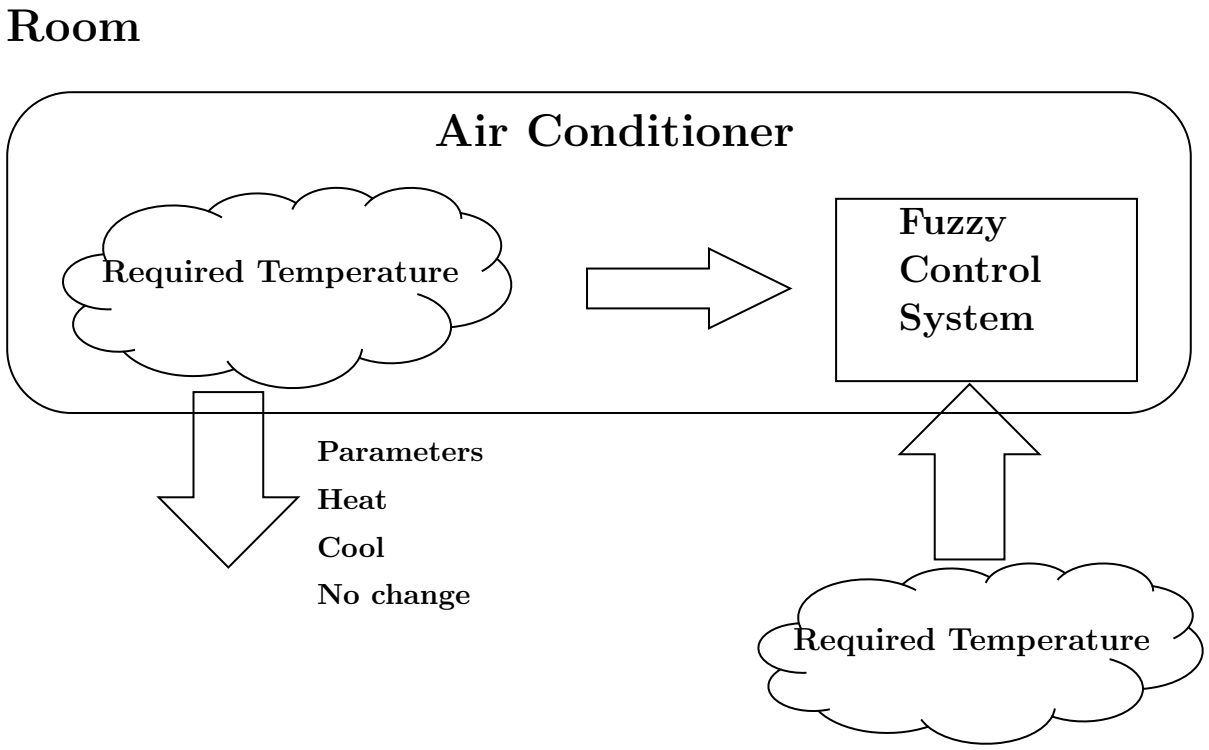
δ={D} Define tabular matrix of the fuzzy set.

	λ	μ	γ	δ
A	1	0	0	0
B	1	1	0	0
C	0	1	0	0
D	0	1	1	1
E	0	1	0	0
F	0	1	0	0
G	1	0	1	0

Thus, the candidates A, C, E, and F have choice value 1 B, G have choice value 2 and D have choice value 3.

Therefore the company must choose the Candidate D.

Example 4.2. Let us consider an air conditioning system 5 level fuzzy control system. This system adjusts the temperature of air conditioner by comparing the room temperature and the highest temperature required value.



The algorithm of fuzzy control system is mentioned below:

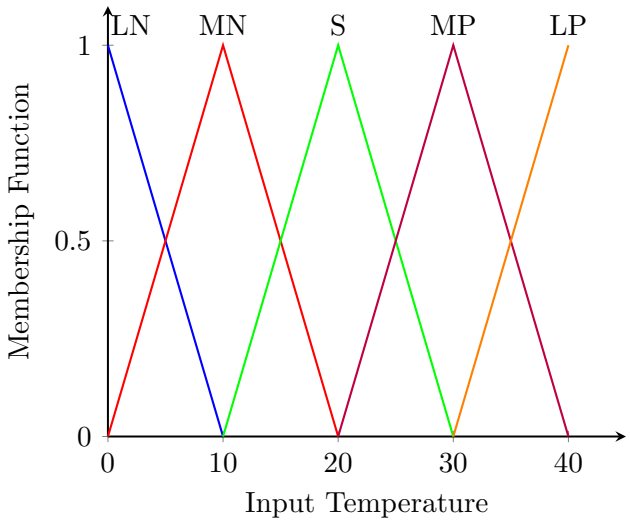
Step 1: Define Linguistic Variable and meaning

Linguistic variables are input and output variables. For room temperature, cold, warm, heat are linguistic terms.

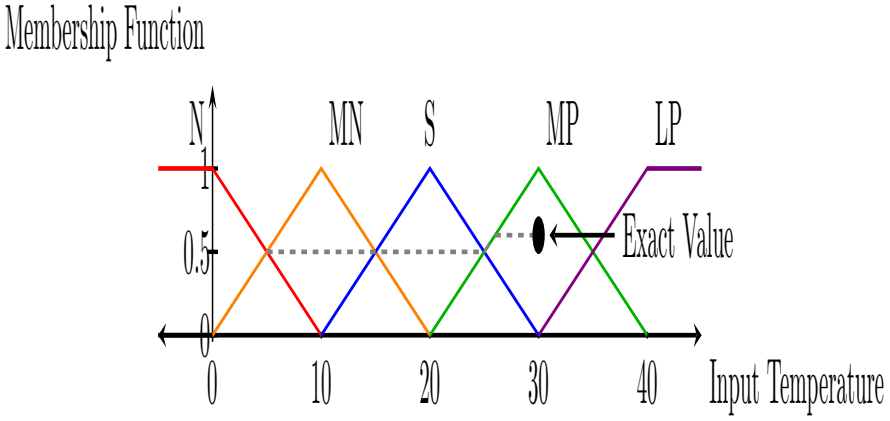
Step 2: To construct membership function Step 3: Construct room temperature base rules

Room temperature required	Heavy Cold	Cold	Warm	Heat	Very Heat
Heavy Cold	——	Heat	Heat	Heat	Heat
Cold	Cool	——	Heat	Heat	Heat
Warm	Cool	Cool	——	Heat	Heat
Heat	Cool	Cool	Cool	——	Heat
Very Heat	Cool	Cool	Cool	Cool	——

S.No	Condition	Sloution
1.	Temperature=Cold (or) very cold	Heat
2.	Temperature=Heat (or) very Heat	Cool
3.	Temperature=Warm	No change



Conclusion:



Note:

Step	Description
LP	X value is large positive
MP	X value is medium positive
S	X value is small
MN	X value is Medium negative
LN	X value is large negative

Example 4.3. We derive the Room Temperature control system

Input: Temperature (°C)

Output: Fan speed (RPM)

Linguistic terms

I. Temperature ranges

1. Low (0-20°C)

2. Medium (15-30°C)

3. High (25-40°C)

II. Fan Speed level 1.Slow

2.Medium

3.Fast

Rules:

1. If temperature is low then fan speed is slow.

2. If temperature is medium then fan speed is medium.

3. If temperature is high then fan speed is fast.

Input

Temperature =26°C $\longrightarrow$ falls partially into "medium" and "high".

Output

Fan speed=1600RPM

Fan Speed(RPM)

1) Slow : 200 - 600

2) Medium : 600 - 1200

3) Fast : 1200 - 2000.

Membership Functions:

Temperature = 26°C

μ - low(26)=0

μ - medium(26)=0.4

μ - high(26)=0.6

Apply Rules:

Rule 1. Not active (μ=0)

Rule 2. Activated with 0.4 - medium speed

Rule 3. Activated with 0.6 - high speed

Calculation:

1) Medium speed (600 - 1200 RPM) μ=0.4 at 1200(RPM).

2) High speed (1200 - 2200 RPM) μ=0.6 at 1600(RPM).

Example 4.4. Fuzzy decision theory in Medical treatment.

Question: Fuzzy decision for diagnosing fever.

1. Body temperature(°C)

2. Cough Intensity (Scale 0-10).

Step 1: Define fuzzy sets

Input 1: Temperature(T)

we define a normal and high temperature in fuzzy sets by

$$\alpha_{Normal}(T) = \begin{cases} 1 & T \leq 36 \\ \frac{37-T}{1} & 36 < T < 37 \\ 1 & T \geq 37 \end{cases} \tag{1}$$

$$\alpha_{Normal}(T) = \begin{cases} 0 & T \leq 37 \\ \frac{T-37}{1} & 37 < T < 40 \\ 1 & T \geq 39 \end{cases} \tag{2}$$

Input 2: Cough Intensity(C)

we define a mild and severe cough in fuzzy sets by

$$\gamma_{mild}(C) = \begin{cases} 1 & C \leq 3 \\ \frac{5-C}{2} & 3 < C < 5 \\ 1 & C \geq 5 \end{cases} \tag{3}$$

$$\eta_{severe}(C) = \begin{cases} 0 & C \leq 4 \\ \frac{C-4}{2} & 4 < C < 6 \\ 1 & C \geq 6 \end{cases} \tag{4}$$

Step 2: Taking Fuzzy Rules

S. No Rules

L1 If temperature is high and cough is severe then fever is severe (0.9)

L2 If temperature is high and cough is mild then fever is average (0.6)

L3 If temperature is high and cough is mild then fever is average (0.2)

Step 3: Taking Fuzzy Rules

1. T=39°C
2. C=4.3(out of 10)

Step 4: We define membership values in the above data

$\alpha_{Normal}(39) = 0$

$\beta_{High}(39) = \frac{39-37}{2} = 1.0$

$\gamma_{Mild}(4.3) = \frac{5-4.3}{2} = 0.35$

$\eta_{severe}(4.3) = \frac{4.3-4}{2} = \frac{0.3-2}{2} = 0.15$

$L_1(severe) : min(1.0, 0.15) = 0.15$

$L_2(average) : min(1.0, 0.35) = 0.35$

$L_3(none) : min(0, 0.35) = 0$

Average for fever $= \frac{0.15*0.9+0.35*0.6}{0.15+0.35} = \frac{0.135+0.21}{0.50} = \frac{0.345}{0.50} = 0.69$

Conclusion : Fever symptoms =0.69 (medium fever but not severe)

Example 4.5. Fuzzy decision making in traffic signal

Question: Control the traffic using green light duration.

(a) Traffic density (vehicles per lane)

(b) Waiting time (in seconds) Step 1: Define fuzzy sets

a) Traffic Density (D) i) Low

$$\lambda_{low}(D) = \begin{cases} 1 & D \leq 15 \\ \frac{25-D}{5} & 5 < D < 25 \\ 0 & D \geq 25 \end{cases} \tag{5}$$

ii) High

$$\mu_{high}(D) = \begin{cases} 0 & D \leq 20 \\ \frac{D-20}{5} & 20 < D < 30 \\ 1 & D \geq 30 \end{cases} \tag{6}$$

b) Waiting Time (w) i)Minimum

$$\alpha_{min}(W) = \begin{cases} 1 & W \leq 20 \\ \frac{60-W}{30} & 20 < D < 60 \\ 0 & W \geq 60 \end{cases} \tag{7}$$

$$\beta_{max}(W) = \begin{cases} 0 & W \leq 50 \\ \frac{W-50}{30} & 50 < W < 30 \\ 1 & W \geq 80 \end{cases} \tag{8}$$

Step 2: Fuzzy Rules

• Rule 1: If Density is high and waiting time is maximum, then green light time is maximum (90 seconds).

• Rule 2: If Density is low and waiting time is minimum, then green light time is minimum (40 seconds).

• Rule 3: If Density is high and waiting time is minimum, then green light time is moderate (60 seconds).

Step 3: Input Data

a) Traffic density = 20 vehicles

b) Waiting time = 63 seconds

Step 4: Construct Membership Values

$\mu_{\text{low}}(20) = \frac{25-20}{5} = \frac{5}{5} = 1$

$\mu_{\text{high}}(20) = \frac{20-20}{5} = 0$

$\alpha_{\min}(63) = \frac{63-60}{30} = \frac{3}{30} = 0.1$

$\beta_{\max}(63) = \frac{63-50}{30} = \frac{13}{30} = 0.43$

Step 5: Discussion Rules

Rule 1: (High, max) : min(0, 0.43) = 0

Rule 2: (low, min) : min(1, 0.1 = 0.1

Rule 3: (high, min) = min(0, 0.1)=0

Step 6

Average for green light time dutation = $\frac{0\times90+0+1\times40+0\times60}{0+0.1+0} = \frac{40}{0.1} = 40$ seconds.

Conclusion: Green light time duration is 40 seconds. (traffic control)

Example 4.6. The new coach of a Cricket team is not sure yet about the quality of he players. He characterized, therefore, the good players as follows.P_1 and P_2 by (1,0), P_3, P_4 and P_5 by (0.7, 0.2),P_6, P_7 and P_8 by (0.6,0.3), P_8, P_9 and P_{10} by (0.7, 0.4),P_{11} by (0.5,0.1) $P_{12}$and P_{13} by (0.6,0.3), P_{14} by (0.3,0.6), P_{15} and P_{16} by (0.3,0.9), P_{17} by (0.1,0.8) and the remaining three players[P_{18}, P_{19} and P_{20}] by (0,1). In other words, the coach believes that $P_1 and P_2$ are good players, he is 80%. P_3, P_4 and P_5 are good players too, but he has 10%. may not be good players and a 10% hesitation to decide about it, and so on. For the last three players the coach absolutely sure that they are not good players. Estimate the overall quality of the team according to the opinion of the coach about the individual quality of his players.

Solution:

The overall quality of the cricket team can be estimated by Calculating the Average values say A: (m,n) to the elements of the Sorry set of the good players of the team.

A = (m,n) = 1/20 [2(1,0)+3(0.7,0.2)+3(0.6,0.3)+3(0.7,0.4)+1(0.5,0.1)+ 2(0.6,0.3) + 1(0.3,0.6) + 2(0.3,0.9) + 1(0.1,0.8)+3(0,1)]

1/20 (9.9,7.3) = (0.495,0.365)

A random player of the team is a good player by 49.5%. but there is a 36.5%. chance to be not a good player and a 14%. hesitation for deciding whether he is a good player or not these outcomes give a quite good idea about the overall quality of the Cricket team.

5. **Conclusion**

Applications of fuzzy Baire spaces extend across various fields such as decision theory, control fuzzy systems, data analysis and artificial Intelligence, where handling imprecise information is crucial. There role to the development of fuzzy topology, fuzzy function spaces and generalized compactness further highlights their importance in both theoretical and applied mathematics. Overall, fuzzy Baire spaces contribute significantly to the advancement of fuzzy set theory and its integration into real world Problem solving Contexts.

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Fuzzy Directed and Weakly Directed Baire Spaces

E. Poongothai, **M. Nalini**

Abstract. We study fuzzy directed Baire spaces and weakly directed Baire spaces on fuzzy topological spaces. A characterization of fuzzy directed Baire spaces is given using fuzzy G_δ- sets. Further, we prove that the weakly directed Baire spaces with a directed Baire spaces. Also we discuss several characterizations of other fuzzy spaces namely fuzzy P- space, fuzzy hyperconnected space.

Keywords: Fuzzy G_δ- set, fuzzy nowhere dense set, fuzzy directed Baire space, fuzzy weakly directed Baire space, fuzzy P- space, fuzzy hyperconnected space.

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1. Introduction

The concepts of Fuzzy sets were introduced by L.A.Zadeh in 1965 [18]. Then the fuzzy set theory is extension by many researchers. The important concept of fuzzy topological space was offered by C. L. Chang [6] and from that point forward different ideas in topology have been reached out to fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

In 1899, Rene Louis Baire [2] introduced the concepts of first and second category sets in his doctoral thesis. In classical topology, Baire space, named in honor of Rene Louis Baire, was first introduced in Bourbaki's [4] Topological generale Chapter IX. The concepts of Baire spaces have been studied extensively in fuzzy topology in [10].

In this paper, we introduce the concept of fuzzy directed Baire space and fuzzy weakly directed Baire spaces. Also, we discuss several characterizations and inter-relations between fuzzy P- space and fuzzy hyperconnected space. Several examples are given to these concepts introduced in this paper.

2. Preliminaries

In order to make the exposition self-contained, we give some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I [18].

Definition 2.1. [6] Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then

(i) $\lambda \vee \mu : X \to [0, 1]$ is defined as follows : $(\lambda \vee \mu)(x) = \text{Max}\{\lambda(x), \mu(x)\}$.

(ii) $\lambda \wedge \mu : X \to [0, 1]$ is defined as follows : $(\lambda \wedge \mu)(x) = \text{Min}\{\lambda(x), \mu(x)\}$.

(iii) $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$.

More generally, for a family $\{\lambda_i : i \in I\}$ of fuzzy sets in (X, T), $\psi = \vee_i \lambda_i$ and $\delta = \wedge_i \lambda_i$ are defined respectively as $\psi(x) = sup_i\{\lambda_i(x) : x \in X\}$ and $\delta(x) = inf_i\{\lambda_i(x) : x \in X\}$.

Definition 2.2. [6] Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T).The interior int(λ) and the closure cl(λ) are defined respectively as follows:

(i). $\text{int}(\lambda) = \vee \{\mu / \mu \leq \lambda, \mu \in T\}$.

(ii). $\text{cl}(\lambda) = \wedge \{\mu / \lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.3. *[1] For a fuzzy set λ of a fuzzy topological space X,*

(i). $1 - int(\lambda) = cl(1 - \lambda)$,

(ii). $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.4. [12] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.

Definition 2.5. [12] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is., int[cl(λ)] = 0, in (X, T).

Definition 2.6. [3] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy G_δ -set in (X, T) if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in$ T.

Definition 2.7. [3] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ -set in (X, T) if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in$ T.

Definition 2.8. [12] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.9. [15] Let λ be a fuzzy first category set in a fuzzy topological space (X, T). Then 1 - λ is called a fuzzy residual set in (X, T).

Definition 2.10. [12] A fuzzy topological space (X, T) is called fuzzy first category if the fuzzy set 1_X is a fuzzy first category set in (X, T). That is, $1_X = \bigvee_{i=1}^{\infty}(\lambda_i)$ where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Otherwise (X, T) will be called a fuzzy second category space .

Lemma 2.11. *[1] For a family A = $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X,* $\vee\ (cl(\lambda_\alpha)) \leq cl(\vee\ (\lambda_\alpha))$. *In case A is a finite set,* $\vee\ (cl(\lambda_\alpha)) = cl(\vee\ (\lambda_\alpha))$. *Also* $\vee\ (int(\lambda_\alpha)) \leq int(\vee\ (\lambda_\alpha))$.

Definition 2.12. [14] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy σ- nowhere dense set, if λ is a fuzzy F_σ-set in (X, T) such that int(λ) = 0.

Definition 2.13. [15] Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if $int[\bigvee_{i=1}^{\infty}(\lambda_i)] = 0$,where (λ_i)'s are fuzzy nowhere dense sets in (X, T).

Definition 2.14. [5] A fuzzy topological space (X, T) is said to be fuzzy hyper-connected if every non-null fuzzy open subset of (X, T) is fuzzy dense set in (X, T). That is a fuzzy topological space (X, T) is hyper-connected if cl(μ_i) = 1, for all $\mu_i \in T$.

Definition 2.15. [17] A fuzzy topological space (X, T) is called a fuzzy P- space if each fuzzy G_δ - set in (X, T) is fuzzy open set in (X, T).

3. Fuzzy Directed and Weakly Directed Baire Spaces

Definition 3.1. Let (X, T) be a fuzzy topological space. Then (X,T) is called a fuzzy directed Baire space if $(\bigvee_{i=1}^{\infty} int(\lambda_i)) = 0$, where (λ_i)'s are fuzzy G_δ - sets and fuzzy nowhere dense sets in (X, T).

Example 3.2. Let X=*{a, b, c}* and the fuzzy sets *α, β, γ, θ,* λ and μ defined on X as follows

α: X → [0,1] is defined by α(a)=0.3, α(b)=0.6, α(c)=0.5

β: X → [0,1] is defined by β(a)=0.4, β(b)=0.7, β(c)=0.3

γ: X → [0,1] is defined by γ(a)=0.8, γ(b)=0.5, γ(c)=0.4

θ: X → [0,1] is defined by θ(a)=0.6, θ(b)=0.7, θ(c)=0.8

λ: X → [0,1] is defined by λ(a)=0.7, λ(b)=0.6, λ(c)=0.7

μ: X → [0,1] is defined by μ(a)=0.8, μ(b)=0.6, μ(c)=0.7

Then T={0, α, β, γ, $\lambda \vee \beta$, $\beta \vee \gamma$, $\alpha \vee \gamma$, $\alpha \wedge \beta$, $\beta \wedge \gamma$, $\alpha \wedge \gamma$, $\alpha \wedge \beta \wedge \gamma$, 1} and T={0, λ, μ, $\theta \vee \lambda$, $\theta \vee \mu$, $\theta \wedge \lambda$, 1} is a fuzzy topology on X. Then $intcl(1 - \mu) = int(1 - \mu) = 0$, $intcl(1-\theta \vee \lambda) = int(1-\theta \vee \lambda) = 0$. Therefore the fuzzy nowhere dense sets are $1-\mu$, $1-(\theta \vee \lambda)$. There fuzzy sets are $(\bigvee_{i=1}^{\infty}(\alpha \wedge (\alpha \wedge \beta) \wedge (\alpha \wedge \gamma)) = \alpha \wedge \beta \wedge \gamma$ is a fuzzy G_δ -set in (X,T). This implies that $int[(\bigvee_{i=1}^{\infty}(\alpha \wedge \beta \wedge \gamma) \vee (1 - \mu) \vee (1 - \theta \vee \lambda)] = 0$. Hence (X,T) is a fuzzy directed Baire Space.

Definition 3.3. Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy weakly directed Baire space if $(\bigvee_{i=1}^{\infty}(\lambda_i)) \neq 0$, where (λ_i)'s are fuzzy G_δ - sets and fuzzy nowhere dense sets in (X, T).

Example 3.4. Let X={*a, b, c*} and the fuzzy sets λ, μ, γ, α, β *and* θ defined on X as follows

λ: X → [0,1] is defined by λ(a)=0.6, λ(b)=0.7, λ(c)=0.5

μ: X → [0,1] is defined by μ(a)=0.6, μ(b)=0.6, μ(c)=0.8

γ: X → [0,1] is defined by γ(a)=0.7, γ(b)=0.7, γ(c)=0.6

α: X → [0,1] is defined by α(a)=0.6, α(b)=0.7, α(c)=0.6

β: X → [0,1] is defined by β(a)=0.6, β(b)=0.5, β(c)=0.6

θ: X → [0,1] is defined by θ(a)=0.7, θ(b)=0.6, θ(c)=0.7

Then T={0, λ, μ, γ, $\lambda \vee \mu$, $\lambda \wedge \mu$, 1} and T= {0, α, β, θ, $\alpha \vee \beta$, $\beta \vee \theta$, $\alpha \vee \theta$, $\alpha \wedge \beta$, $\beta \wedge \theta$, 1}. Then $intcl(1 - \alpha) = int(1 - \alpha) = 0$ $intcl(1 - \beta) = int(1 - \beta) = 0$. The fuzzy sets are $\bigvee_{i=1}^{\infty}[(\lambda \wedge \mu) \wedge (\mu \wedge \gamma)] = \lambda \wedge \mu$ is a fuzzy G_δ- sets in (X, T). This implies that int $(\bigvee_{i=1}^{\infty}(\lambda \wedge \mu) \vee (1 - \alpha) \vee (1 - \beta)) = \alpha \wedge \beta = 0$. Hence (X,T) is a fuzzy directed Baire Space.

Proposition 3.5. *Let (X,T) be a fuzzy topological space. Then the following are equivalent.*

(i) (X,T) is a fuzzy directed Baire space.

(ii) int(λ) = 0, for every fuzzy first category set λ in (X,T).

(iii) cl(μ) = 1,for each fuzzy residual set μ in (X,T).

Proof. (i) $\Rightarrow$ (ii)

Let λ be a fuzzy first category set in (X,T). Then $\lambda_i = (\bigvee_{i=1}^{\infty}(\lambda i))$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Then $int(\lambda) = int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$. Since (X,T) is a fuzzy directed Baire space, $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$. Hence $int(\lambda) = 0$ for fuzzy first category set λ in (X,T).

(ii) $\Rightarrow$ (iii)

Let μ be a fuzzy residual set. Then $(1 - \mu)$ is a fuzzy first category set in (X,T). By hypothesis, $int(1-\mu) = 0$. Then by Lemma (2.3) $1 - cl(\mu) = 0$. Hence $cl(\mu) = 1$, for fuzzy residual set μ in (X,T).

(iii) $\Rightarrow$ (i)

Let λ be a fuzzy first category set in (X,T). Then $\lambda = (\bigvee_{i=1}^{\infty}(\lambda_i))$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Now λ is a fuzzy first category set implies that $(1 - \lambda)$ is a fuzzy residual set in (X,T). By hypothesis, we have $cl(1 - \lambda) = 1$ then by Lemma(2.3), $1 - int(\lambda) = 1$. Hence $int(\lambda) = 0$. That is, $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Hence (X,T) is a fuzzy directed Baire space.

Proposition 3.6. *If the fuzzy topological space (X,T) is a fuzzy directed Baire space. Then (X,T) is a fuzzy second category space.*

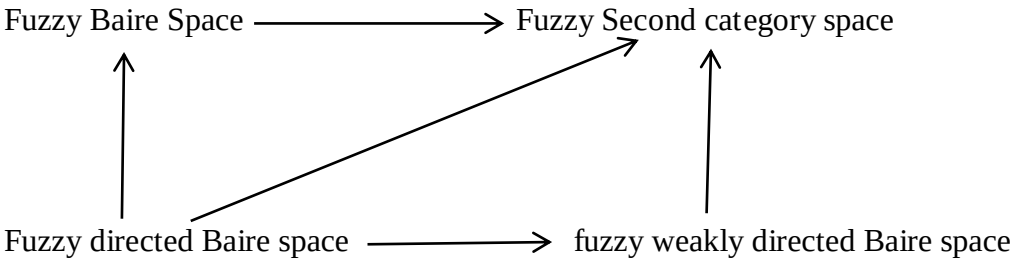
Proof. Let (X,T) be a fuzzy directed Baire space. Then $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are G_δ - sets and fuzzy nowhere dense set in (X,T). Then $\bigvee_{i=1}^{\infty}(\lambda_i) \neq 1$ (otherwise $\bigvee_{i=1}^{\infty}(\lambda_i) = 1$ implies that $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = int(1) = 1$ and hence $0 = 1$, a contradiction). Hence (X,T) is fuzzy second category space.

Proposition 3.7. *If the fuzzy P - space (X,T) is a fuzzy hyperconnected space, then (X,T) is a fuzzy directed Baire space.*

Proof. Let λ be a fuzzy G_δ - set in a fuzzy P- space (X,T). Since (X,T) is a fuzzy P-space is a fuzzy open set in (X,T). Since the fuzzy P- space (X,T) is a fuzzy hyper connected space, then fuzzy open set λ in (X,T) is a fuzzy dense set in (X,T).That is $cl(\lambda) = 1$. Hence λ is a fuzzy G_δ- set and fuzzy nowhere dense set in (X,T). Then by the proposition[3.5], $(1 - \lambda)$ is a fuzzy first category set in (X,T). Therefore $(1 - \lambda) = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Then, $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = int(1 - \lambda) = 1 - cl(\lambda) = 1 - 1 = 0$. Hence we have $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are

fuzzy nowhere dense sets in (X,T). Also, λ is a fuzzy G_δ - sets in (X,T). Therefore (X,T) is a fuzzy directed Baire space.

The following arrow diagram shows the relation between the four spaces namely, fuzzy Baire space, fuzzy second category, fuzzy directed Baire space, fuzzy weakly directed Baire space.



Proposition 3.8. *If the fuzzy first category set λ in a fuzzy directed Baire space (X,T) is a fuzzy closed set, Then int(cl(λ)) = 0 in (X,T).*

Proof. Let λ be a fuzzy first category in a fuzzy topological space (X,T). Since (X,T) fuzzy directed Baire space, by Proposition 3.5, int(λ)=0. Now λ is a fuzzy closed set int (X,T), implies that cl(λ)=λ. Hence int cl(λ) = int(λ) = 0. Therefore int(cl(λ)) = 0 in (X,T).

Proposition 3.9. *If λ is any fuzzy first category set and G_δ - set in a fuzzy Baire space (X,T). Such that cl(λ) = λ , then (X,T) is a fuzzy directed Baire Space.*

Proof: The proof follows from the above.

Proposition 3.10. *If λ be a fuzzy set in a fuzzy directed Baire space (X,T) then 1-λ is fuzzy first category set in (X,T).*

Proof. Let λ be a fuzzy set in a fuzzy directed Baire space (X,T). Since λ is a intersection of fuzzy G_δ - sets in (X,T), $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i) \in T$ and since λ is a fuzzy dense set in (X,T), cl(λ) = 1. Then cl($\bigvee_{i=1}^{\infty}(\lambda_i)$) = 1. But cl($\bigvee_{i=1}^{\infty}(\lambda_i)$) $\leq$ ($\bigvee_{i=1}^{\infty}$ *cl*(λ_i)). Hence 1 $\leq \bigvee_{i=1}^{\infty}$ *cl*(λ_i). That is, $\bigvee_{i=1}^{\infty}$ *cl*(λ_i) = 1. Then we have cl(λ_i) = 1, for each $\lambda_i \in T$ and hence cl(int(λ_i)) =1, which implies that 1 - cl(int(λ_i))=0 and hence intcl(1$-\lambda_i$))=0. Therefore 1-λ_i is a fuzzy nowhere dense set in (X,T). Now 1-λ = 1-$\bigvee_{i=1}^{\infty} \infty(\lambda i) = \bigvee_{i=1}^{\infty}(1 - \lambda_i)$. Therefore 1-$\lambda = \bigvee_{i=1}^{\infty}(1 - \lambda_i)$ where (1-λ_i)'s are fuzzy nowhere dense sets in (X,T). Hence 1-λ is a first category set in (X,T).

Proposition 3.11. *If λ be a fuzzy set in a fuzzy directed Baire space (X,T) then λ is fuzzy residual set in (X,T).*

Proof. Let λ be a fuzzy set in a fuzzy directed Baire space (X,T), since intersection of fuzzy G_δ - sets is fuzzy dense set in (X,T). By Prop 3.5, we have 1-λ is a fuzzy first category set in (X,T) and hence λ is fuzzy residual set in (X,T).

Proposition 3.12. *If the fuzzy topological space (X,T) is a fuzzy Directed Baire space, then (X,T) is a fuzzy Baire space.*

Proof. Let λ be a fuzzy set in a fuzzy directed Baire space (X,T), then λ is a intersection of fuzzy G_δ- sets is fuzzy dense set in (X,T). By Prop [3.5], 1-λ is a fuzzy first category set in (X,T) and $1-\lambda = (\bigvee_{i=1}^{\infty}(1-\lambda_i))$, where $(1-\lambda_i)$'s are fuzzy nowhere dense set in (X,T). But $int(1-\lambda)$ = 1-cl(λ) =1-1=0. (Since λ is fuzzy dense),cl(λ)=1. Then $int(\bigvee_{i=1}^{\infty}(1-\lambda_i)) = int(1-\lambda) = 0$ and hence (X,T) is a fuzzy Baire space.

Proposition 3.13. *If the fuzzy topological space (X,T) is a fuzzy Directed Baire space, then* $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, *where* (λ_i)*'s are fuzzy dense and fuzzy open sets in (X,T).*

Proof. Let λ be a fuzzy set in a fuzzy directed Baire space (X,T). Then $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy open sets in (X,T). Now, $cl(\lambda) = cl[\bigvee_{i=1}^{\infty}(\lambda_i)]$ in (X,T). But $cl[\bigvee_{i=1}^{\infty}(\lambda_i)] \leq [\bigvee_{i=1}^{\infty} cl(\lambda_i)]$, implies that $1 \leq [\bigvee_{i=1}^{\infty} cl(\lambda_i)]$. That is, $\bigvee_{i=1}^{\infty} cl(\lambda_i) = 1$ and then $cl(\lambda_i) = 1$ in (X,T). Hence $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy dense and fuzzy open sets in (X,T).

Proposition 3.14. *If* λ *is a fuzzy set in a fuzzy Directed Baire space (X, T) such that* $\mu \leq 1-\lambda$, *where* μ *is a fuzzy* F_σ *- set in (X, T), then* μ *is a fuzzy* σ *- nowhere dense set in (X, T).*

Proof. Let λ be a fuzzy set in a fuzzy directed Baire space. Since intersection fuzzy G_δ- sets in (X,T), is fuzzy dense set, This implies that λ is a fuzzy dense set in (X, T) such that $\mu \leq 1-\lambda$. Now $\mu \leq (1-\lambda) \Rightarrow int(\mu) \leq int(1-\lambda) = 1 - cl(\lambda) = 1 - 1 = 0$ and hence $int(\mu) = 0$. Therefore μ is a fuzzy σ - nowhere set in (X, T).

Proposition 3.15. *If* λ *is a fuzzy set in a fuzzy Directed Baire space (X, T) such that* $\mu \leq 1-\lambda$, *where* μ *is a fuzzy* F_σ *- set in (X,T), then* μ *is a fuzzy* σ *- nowhere dense set in (X, T). Then (X, T) is a fuzzy* σ *- Baire space.*

Proof. The above Prof 3.14, Let μ is a fuzzy σ nowhere dense set in (X, T). Hence (X,T) is a fuzzy σ - Baire space.

Theorem 3.16. *[10] In a fuzzy topological space (X,T), a fuzzy set* λ *is nowhere dense set if and only if* $1-\lambda$ *is a fuzzy dense and fuzzy* G_δ*- set in (X, T).*

Proposition 3.17. *If a fuzzy topological space (X,T) is a fuzzy Volterra space, then* $int(\bigvee_{i=1}^{N}(\lambda_i)) = 0$, *where* (λ_i)*'s are fuzzy nowhere dense sets in (X,T).*

Proof. Let (λ_i)'s [i=1 to N] be fuzzy nowhere dense sets in (X,T). Then by Theorem 3.16 $(1-\lambda_i)$'s are fuzzy dense and G_δ- sets in (X,T). Since (X,T) is a fuzzy Volterra space, $cl(\bigwedge_{i=1}^{N}(1-\lambda_i)) = 1$. Now $cl(\bigwedge_{i=1}^{N}(1-\lambda_i)) = 1$ implies that $cl(1-\bigvee_{i=1}^{N}(\lambda_i)) = 1$. Then we have $1 - int(\bigvee_{i=1}^{N}(1-\lambda_i)) = 1$. This implies $int(\bigvee_{i=1}^{N}(\lambda i)) = 0$.

Proposition 3.18. *If a fuzzy topological space (X,T) is a fuzzy Directed Baire space, then (X,T) is a fuzzy Volterra space.*

Proof. Let (X,T) be a fuzzy directed Baire Space. then $int(\bigvee_{i=1}^{N}(\lambda_i)) = 0$ where (λ_i)'s are fuzzy G_δ- sets and fuzzy nowhere dense sets in (X,T). Now $int(\bigvee_{i=1}^{N}(\lambda_i)) \leq int(\bigvee_{i=1}^{\infty}(\lambda_i))$ implies that $int(\bigvee_{i=1}^{N}(\lambda_i)) = 0$. Then $1-int(\bigvee_{i=1}^{N}(\lambda_i)) = 1-0 = 1$. Then we have $cl(1-\bigvee_{i=1}^{N}(\lambda_i)) = 1$. This implies that $cl(\bigwedge_{i=1}^{N}(\lambda_i)) = 1$. Since (λ_i)'s [i=1 to N] are fuzzy G_δ - sets and fuzzy nowhere dense sets in (X,T) by Theorem 3.16 $(1-\lambda_i)$'s are fuzzy dense and fuzzy G_δ - sets in (X,T). Hence $cl(\bigwedge_{i=1}^{N}(1-\lambda_i)) =$, where $(1-\lambda_i)$'s are fuzzy dense and fuzzy G_δ - sets in (X,T) implies that (X,T) is a fuzzy Volterra space.

Remark 3.19. A fuzzy Baire space need not be a fuzzy Directed Baire space. For consider, the following example:

λ: X $\rightarrow$ [0,1] is defined by $\lambda(a)=0.8$, $\lambda(b)=0.6$, $\lambda(c)=0.7$

μ: X $\rightarrow$ [0,1] is defined by $\mu(a)=0.6$, $\mu(b)=0.9$, $\mu(c)=0.8$

γ: X $\rightarrow$ [0,1] is defined by $\gamma(a)=0.7$, $\gamma(b)=0.5$, $\gamma(c)=0.9$

α: X $\rightarrow$ [0,1] is defined by $\alpha(a)=0.6$, $\alpha(b)=0.5$, $\alpha(c)=0.5$

β: X $\rightarrow$ [0,1] is defined by $\beta(a)=0.5$, $\beta(b)=0.4$, $\beta(c)=0.4$

θ: X $\rightarrow$ [0,1] is defined by $\theta(a)=0.4$, $\theta(b)=0.5$, $\theta(c)=0.4$

Then T=$\{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\lambda \wedge \gamma), (\lambda \vee \gamma), (\mu \wedge \gamma), \lambda \wedge (\mu \vee \gamma), \lambda \vee (\mu \wedge \gamma), \mu \wedge (\lambda \vee \gamma), \mu \vee (\lambda \wedge \vee), \gamma \wedge (\lambda \vee \mu), \gamma(\lambda \wedge \mu), (\lambda \wedge \mu \wedge \gamma), (\lambda \vee \mu \vee \gamma), 1\}$ is a fuzzy topology on X. Then T=$\{0, \alpha, \beta, \delta, (\alpha \vee \beta), (\beta \vee \theta), (\alpha \wedge \beta), (\beta \wedge \theta), \theta \wedge (\alpha \vee \beta), 1\}$ Now, $(1-\lambda)$, $(1-\mu)$, $(1-\gamma)$, $(1-\lambda \vee \mu)$ are fuzzy nowhere dense sets and $\alpha \vee \beta$, $\beta \vee \theta$ are fuzzy G_δ- sets in (X,T). Then $\delta = \{(1-\lambda) \vee (1-\mu) \vee (1-\gamma) \vee (\alpha \vee \beta)\} \neq 0$, and $\eta = int[(1-\lambda) \vee (1-\mu) \vee (1-\gamma) \vee (1-\lambda \vee \mu)] = 0$. Hence (X,T) is not a fuzzy directed Baire space whereas (X,T) is a fuzzy Baire space.

Proposition 3.20. *If the fuzzy topological space (X,T) is a fuzzy first category space, Then (X,T) is not a fuzzy directed Baire space.*

Proof. Let (X, T) is a fuzzy first category space. Then ($\bigvee_{i=1}^{\infty}(\lambda_i)$) = 1_x , where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Now for the fuzzy first category set 1_x, we have $intcl(1_x) \neq 0$. Hence (X,T) is not a fuzzy directed Baire space.

Theorem 3.21. *[9] If λ be a fuzzy dense and fuzzy G_δ- set in a fuzzy topological space (X,T), then ($1 - \lambda$) is a fuzzy first category set in (X, T).*

Proposition 3.22. *If a fuzzy topological space(X, T) has a fuzzy dense and fuzzy G_δ- set, then (X, T) is not a fuzzy directed Baire space.*

Proof. Let λ be a fuzzy dense and fuzzy G_δ- set in (X, T). Then by Theorem 3.21 , (1 - λ)is a first category set in (X, T). Now $intcl(1 - \lambda) = 1 - clint(\lambda) > 1 - cl(\lambda) > 1 - 1 = 0$ (Since $cl(\lambda) = 1$). Hence we have intcl$(1 - \lambda) \neq 0$, for a fuzzy first category set $(1 - \lambda)$ in (X,T). Therefore (X,T) is not a fuzzy directed Baire space.

Proposition 3.23. *If every fuzzy residual set in a fuzzy P-space (X,T) contains a non-zero fuzzy dense and fuzzy G_δ - set in (X,T), then (X,T) is a fuzzy Directed Baire space.*

Proof. Let λ be a fuzzy first category in (X,T). Then $(1 - \lambda)$ is a fuzzy residual set in (X,T). Then there exists a fuzzy G_δ - set μ in (X,T) such that $\mu \leq (1 - \lambda)$. Then $\lambda \leq (1 - \mu)$. Now $intcl(\lambda) \leq intcl(1 - \mu)$, implies that $intcl(\lambda) \leq 1 - clint(\mu)$. Since (X,T) is a fuzzy P- space, he fuzzy G_δ- set, μ is fuzzy open in (X,T) and $int(\mu) = \mu$. Therefore we have $intcl(\lambda) \leq 1 - cl(\mu) = 1 - 1 = 0$. (Since μ is a fuzzy dense). Then we have $intcl(\lambda) = 0$ and hence λ is a fuzzy nowhere dense set and fuzzy G_δ- set in (X,T). Hence $int(\bigvee_{i=1}^{\infty}(\lambda i)) = 0$.

Theorem 3.24. *[16] If(X, T) is a fuzzy open hereditarily irresolvable space, any fuzzy σ- nowhere dense set in (X, T) is a fuzzy nowhere dense set in (X, T).*

Proposition 3.25. *If the fuzzy topological space (X, T) is a fuzzy directed Baire space and fuzzy open hereditarily irresolvable space (X,T), then (X, T) is a fuzzy Baire Space.*

Proof. Let (X,T)be a fuzzy directed Baire Space and fuzzy open hereditarily irresolvable space. Then, $int(\bigvee_{i=1}^{\infty}(\lambda i)) = 0$, where ($\lambda_i$)'s are fuzzy G_δ- sets and fuzzy nowhere dense sets in (X,T). Hence $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where ($\lambda_i$)'s are fuzzy nowhere dense sets in (X,T). Therefore (X,T) is a fuzzy Baire space.

4. Conclusion

In this study, we explored the structural and topological properties of fuzzy directed Baire spaces, extending classical Baire space theory into the fuzzy and directed setting. By integrating the concepts of fuzzy topology and directed sets, we were able to generalize key properties such as completeness and category in a more flexible, graded context. Our findings demonstrate that fuzzy directed Baire spaces retain many essential characteristics of traditional Baire spaces while offering a richer framework for handling uncertainty and gradation.

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A note on fuzzy σ-Baire spaces

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ABSTRACT. In this paper we investigate several characterizations of fuzzy σ-Baire spaces and study the conditions under which a fuzzy topological space becomes a fuzzy σ-Baire space.

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Keywords: Fuzzy σ-nowhere dense set, Fuzzy σ-first cateory space, Fuzzy σ-second category space, Fuzzy σ-Baire space, Fuzzy almost resolvable space, Fuzzy hyperconnected space, Fuzzy submaximal space.

1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. ZADEH in his classical paper [16] in the year 1965. This concept provides a natural foundation for treating mathematically the fuzzy phenomena, which exist pervasively in our real world and for new branches of fuzzy mathematics. Thereafter the paper of C.L.CHANG [4] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concepts of Baire spaces have been studied extensively in classical topology in [7], [8], [9] and [6]. The concept of Baire spaces in fuzzy setting was introduced and studied by G.THANGARAJ and S.ANJALMOSE in [11]. The concept of fuzzy σ-Baire spaces was introduced and studied by the authors in [13]. In this paper we investigate several characterizations of fuzzy σ-Baire spaces and study under what conditions a fuzzy topological space becomes a fuzzy σ-Baire space. The inter-relations between fuzzy σ-Baire spaces, fuzzy Baire spaces and fuzzy almost resolvable spaces are also studied in this paper.

2. Preliminaries

By a fuzzy topological space we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X, T).

Definition 2.1. Let λ and μ be any two fuzzy sets in (X, T). Then we define $\lambda \vee \mu : X \to [0, 1]$ as follows : $(\lambda \vee \mu)(x) = Max\{\lambda(x), \mu(x)\}$. Also we define $\lambda \wedge \mu : X \to [0, 1]$ as follows : $(\lambda \wedge \mu)(x) = Min\{\lambda(x), \mu(x)\}$.

Definition 2.2. Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define

(i). $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$,
(ii). $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.1 ([1]). *For any fuzzy set λ in a fuzzy topological space (X, T),*

(a). $1 - cl(\lambda) = int(1 - \lambda)$,
(b). $1 - int(\lambda) = cl(1 - \lambda)$.

Definition 2.3 ([10]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy dense* if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.

Definition 2.4 ([2]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy F_σ-set* in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.5 ([2]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy G_δ-set* in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in T$ for $i \in I$.

Definition 2.6 ([3]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy σ-nowhere dense set* if λ is a fuzzy F_σ-set in (X, T) such that $int(\lambda) = 0$.

Lemma 2.2 ([1]). For a family $\mathscr{A}$ of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space X, $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case $\mathscr{A}$ is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$.

Definition 2.7 ([3]). A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy σ-first category* if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of *fuzzy σ-second category.*

Definition 2.8 ([3]). Let λ be a fuzzy σ-first category set in a fuzzy topological space (X, T). Then $1 - \lambda$ is called a *fuzzy σ-residual set* in (X, T).

3. Fuzzy σ-Baire spaces

Motivated by the classical concept introduced in [3], the concept of fuzzy σ-Baire spaces was introduced and studied in [13].

Definition 3.1 ([13]). Let (X, T) be a fuzzy topological space. Then (X, T) is called a *fuzzy σ-Baire space* if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where λ_i's are fuzzy σ-nowhere dense sets in (X, T).

Theorem 3.1 ([13]). *Let (X, T) be a fuzzy topological space. Then the following are equivalent:*

(1) *(X, T) is a fuzzy σ-Baire space.*

(2) $int(\lambda) = 0$ *for every fuzzy* σ*-first category set* λ *in* (X, T).
(3) $cl(\mu) = 1$ *for every fuzzy* σ*-residual set* μ *in* (X, T).

Theorem 3.2 ([12]). *If* λ *is a fuzzy dense and fuzzy* G_δ*-set in a fuzzy topological space* (X, T)*, then* $1 - \lambda$ *is a fuzzy first category set in* (X, T).

Theorem 3.3 ([13]). *In a fuzzy topological space* (X, T)*, a fuzzy set* λ *is a fuzzy* σ*-nowhere dense set in* (X, T) *if and only if* $1 - \lambda$ *is a fuzzy dense and fuzzy* G_δ*-set in* (X, T).

Proposition 3.1. *If the fuzzy topological space* (X, T) *is a fuzzy* σ*-Baire space, then* $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$*, where the fuzzy sets* (λ_i)*'s* $(i = 1$ *to* $\infty)$ *are fuzzy dense and fuzzy* G_δ*-sets in* (X, T).

Proof. Let (λ_i)'s $(i = 1$ *to* $\infty)$ be fuzzy dense and fuzzy G_δ-sets in (X, T). By theorem 3.3, $(1 - \lambda_i)$'s are fuzzy σ-nowhere dense sets in (X, T). Then the fuzzy set $\lambda = \vee_{i=1}^{\infty}(1-\lambda_i)$ is a fuzzy σ-first category set in (X, T). Now $int(\lambda) = int(\vee_{i=1}^{\infty}(1-\lambda_i)) = int(1-[\wedge_{i=1}^{\infty}(\lambda_i)]) = 1-cl[\wedge_{i=1}^{\infty}(\lambda_i)]$. Since (X, T) is a fuzzy σ-Baire space,by theorem 3.1, we have $int(\lambda) = 0$. Then $1 - cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 0$. This implies that $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$. □

Proposition 3.2. *If the fuzzy topological space* (X, T) *is a fuzzy* σ*-Baire space, then* $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$*, where the fuzzy sets* $(1 - \lambda_i)$*'s* $(i = 1$ *to* $\infty)$ *are fuzzy first category sets formed from the fuzzy dense and fuzzy* G_δ*-sets* λ_i *in* (X, T).

Proof. Let the fuzzy topological space (X, T) is a fuzzy σ-Baire space and the fuzzy sets (λ_i)'s $(i = 1$ *to* $\infty)$ be fuzzy dense and fuzzy G_δ-sets in (X, T). By proposition 3.1, $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$. Then $1-cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 0$. This implies that $int(\vee_{i=1}^{\infty}(1-\lambda_i)) = 0$. Also by theorem 3.2, $(1 - \lambda_i)$'s are fuzzy first category sets in (X, T). Hence $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where the fuzzy sets $(1 - \lambda_i)$'s $(i = 1$ *to* $\infty)$ are fuzzy first category sets formed from the fuzzy dense and fuzzy G_δ-sets λ_i in (X, T). □

Theorem 3.4 ([11]). *Let* (X, T) *be a fuzzy topological space. Then the following are equivalent:*
(1) (X, T) *is a fuzzy Baire space.*
(2) $int(\lambda) = 0$ *for every fuzzy first category set* λ *in* (X, T).
(3) $cl(\mu) = 1$ *for every fuzzy residual set* μ *in* (X, T).

Proposition 3.3. *If the fuzzy first category sets are formed from the fuzzy dense and fuzzy* G_δ*-sets in a fuzzy* σ*-Baire space* (X, T)*, then* (X, T) *is a fuzzy Baire space.*

Proof. Let the fuzzy topological space (X, T) be a fuzzy σ-Baire space. By proposition 3.2, $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where the fuzzy sets $(1 - \lambda_i)$'s $(i = 1$ *to* $\infty)$ are fuzzy first category sets formed from the fuzzy dense and fuzzy G_δ-sets λ_i in (X, T). Now $\vee_{i=1}^{\infty}[int(1-\lambda_i)] \leq int(\vee_{i=1}^{\infty}[1-\lambda_i])$. Then we have $\vee_{i=1}^{\infty}[int(1-\lambda_i)] = 0$. This implies that $int(1 - \lambda_i) = 0$, where $(1 - \lambda_i)$ is a fuzzy first category set in (X, T). By theorem 3.4, (X, T) is a fuzzy Baire space. □

Definition 3.2 ([13]). A fuzzy topological space (X, T) is called *fuzzy* σ*-first category* if the fuzzy set 1_X is a fuzzy σ-first category set in (X, T). That is, $1_X =$

$\vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X,T). Otherwise, (X,T) will be called a *fuzzy σ-second category space.*

Proposition 3.4. *If the fuzzy topological space (X,T) is a fuzzy σ-first category space, then (X,T) is not a fuzzy σ-Baire space.*

Proof. Let the fuzzy topological space (X,T) is a fuzzy σ-first category space. Then $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X,T). Now $int(\vee_{i=1}^{\infty}(\lambda_i)) = int(1_X) = 1 \neq 0$. Hence by definition, (X,T) is not a fuzzy σ-Baire space. □

Proposition 3.5. *If $\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$, where the fuzzy sets (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in a fuzzy topological space (X,T), then (X,T) is a fuzzy σ-second category space.*

Proof. Let (λ_i)'s ($i = 1$ *to* ∞) are fuzzy dense and fuzzy G_δ-sets in (X,T). By theorem 3.3, $(1-\lambda_i)$'s are fuzzy σ-nowhere dense sets in (X,T). Now $\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$ implies that $1 - \wedge_{i=1}^{\infty}(\lambda_i) \neq 1$. Then $\vee_{i=1}^{\infty}(1-\lambda_i) \neq 1$. Hence (X,T) is not a fuzzy σ-first category space and therefore (X,T) is a fuzzy σ-second category space. □

Proposition 3.6. *If λ is a fuzzy σ-first category set in (X,T), then there is a fuzzy F_σ-set δ in (X,T) such that $\lambda \leq \delta$.*

Proof. Let λ be a fuzzy σ-first category set in (X,T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_i's are fuzzy σ-nowhere dense sets in (X,T). Now $[1 - cl(\lambda_i)]$'s are fuzzy open sets in (X,T). Then $\mu = \wedge_{i=1}^{\infty}(1 - cl(\lambda_i))$ is a fuzzy G_δ-set in (X,T) and $1 - \mu = 1 - [\wedge_{i=1}^{\infty}(1 - cl(\lambda_i))] = \vee_{i=1}^{\infty}(cl(\lambda_i))$. Now $\lambda = \vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty}(cl(\lambda_i)) = 1 - \mu$. That is, $\lambda \leq 1 - \mu$ and $1 - \mu$ is a fuzzy F_σ-set in (X,T). Let $\delta = 1 - \mu$. Hence, if λ is a fuzzy σ-first category set in (X,T), then there is a fuzzy F_σ-set δ in (X,T) such that $\lambda \leq \delta$. □

Proposition 3.7. *If δ is a fuzzy σ-residual set in a fuzzy topological space (X,T) such that $\eta \leq \delta$, where η is a fuzzy dense and fuzzy G_δ-set in (X,T), then (X,T) is a fuzzy σ-Baire space.*

Proof. Let δ be a fuzzy σ-residual set in a fuzzy topological space (X,T). Then $1-\delta$ is a fuzzy σ-first category set in (X,T). Now by proposition 3.6, there is a fuzzy F_σ-set μ in (X,T) such that $1 - \delta \leq \mu$. This implies that $1 - \mu \leq \delta$. Let $\eta = 1 - \mu$. Then η is a fuzzy G_δ-set in (X,T) and $\eta \leq \delta$ implies that $cl(\eta) \leq cl(\delta)$. If $cl(\eta) = 1$, then we have $cl(\delta) = 1$. Hence, by theorem 3.1, (X,T) is a fuzzy σ-Baire space. □

Proposition 3.8. *If the fuzzy topological space (X,T) is a fuzzy σ-Baire space and if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, then there exists atleast one F_σ-set λ_i such that $int(\lambda_i) \neq 0$.*

Proof. Suppose that $int(\lambda_i) = 0$, for $i = 1$ *to* ∞, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X,T). Then $\vee_{i=1}^{\infty}(\lambda_i) = 1$. Implies that $int[\vee_{i=1}^{\infty}(\lambda_i)] = int[1] = 1 \neq 0$, a contradiction to (X,T) being a fuzzy σ-Baire space. Hence $int(\lambda_i) \neq 0$, for atleast one F_σ-set λ_i in (X,T). □

Proposition 3.9. *If the fuzzy topological space (X,T) is a fuzzy σ-Baire space, then no non-zero fuzzy open set is a fuzzy σ-first category set in (X,T).*

Proof. Let λ be a non-zero fuzzy open set in a fuzzy σ-Baire space (X,T). Suppose that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where the fuzzy sets (λ_i)'s are fuzzy σ-nowhere dense sets in (X,T). Then $int(\lambda) = int(\vee_{i=1}^{\infty}(\lambda_i))$. Since (X,T) is a fuzzy σ-Baire space, $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$. This implies that $int(\lambda) = 0$. Then we will have $\lambda = 0$, which is a contradiction, since $\lambda \in T$ implies that $int(\lambda) = \lambda \neq 0$. Hence no non-zero fuzzy open set is a fuzzy σ-first category set in (X,T). □

Definition 3.3 ([2])**.** A fuzzy topological space (X,T) is called a *fuzzy submaximal space* if for each fuzzy set λ in (X,T) such that $cl(\lambda) = 1$, then $\lambda \in T$ in (X,T).

Proposition 3.10. *If the fuzzy topological space (X,T) is a fuzzy submaximal space and if λ is a fuzzy σ-first category set in (X,T), then λ is a fuzzy first category set in (X,T).*

Proof. Let $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ be a fuzzy σ-first category set in (X,T), where the fuzzy sets (λ_i)'s $(i = 1 \text{ to } \infty)$ are fuzzy σ-nowhere dense sets in (X,T). Then we have $int(\lambda_i) = 0$ and (λ_i)'s $(i = 1 \text{ to } \infty)$ are fuzzy F_σ-sets in (X,T). Now $int(\lambda_i) = 0$, implies that $1 - int(\lambda_i) = 1 - 0 = 1$ and hence $cl(1 - \lambda_i) = 1$. Since (X,T) is a fuzzy submaximal space, the fuzzy dense sets $(1-\lambda_i)$'s are fuzzy open sets in (X,T) and hence (λ_i)'s are fuzzy closed sets in (X,T). Then $cl(\lambda_i) = (\lambda_i)$ and $int(\lambda_i) = 0$ implies that $intcl(\lambda_i) = int(\lambda_i) = 0$. That is, (λ_i)'s are fuzzy nowhere dense sets in (X,T). Therefore $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy first category set in (X,T). □

Proposition 3.11. *If the fuzzy topological space (X,T) is a fuzzy σ-Baire space and fuzzy submaximal space, then (X,T) is a fuzzy Baire space.*

Proof. Let μ be a fuzzy σ-first category set in (X,T). Since (X,T) is a fuzzy submaximal space, by proposition 3.10, μ is a fuzzy first category set in (X,T). Since (X,T) is a fuzzy σ-Baire space, by theorem 3.1, $int(\mu) = 0$. Hence, for the fuzzy first category set μ in (X,T), we have $int(\mu) = 0$. Therefore, by theorem 3.4, (X,T) is a fuzzy Baire space. □

Definition 3.4 ([14])**.** A fuzzy topological space (X,T) is called a *fuzzy P-space* if countable intersection of fuzzy open sets in (X,T) is fuzzy open. That is, every non-zero fuzzy G_δ-set in (X,T) is fuzzy open in (X,T).

Proposition 3.12. *If the fuzzy topological space (X,T) is a fuzzy σ-Baire space and fuzzy P-space, then (X,T) is a fuzzy Baire space.*

Proof. Let the fuzzy topological space (X,T) be a fuzzy σ-Baire space. Then, by proposition 3.1, $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, where the fuzzy sets (λ_i)'s $(i = 1 \text{ to } \infty)$ are fuzzy dense and fuzzy G_δ-sets in (X,T). Now from, $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, we have $1 - cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 0$. This implies that $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$. Since the fuzzy sets (λ_i)'s are fuzzy dense in (X,T), $cl(\lambda_i) = 1$. Then we have $1 - cl(\lambda_i) = 0$. This implies that $int(1 - \lambda_i) = 0$. Also, since (X,T) is a fuzzy P-space, the non-zero fuzzy G_δ-sets (λ_i)'s in (X,T), are fuzzy open in (X,T). Then $(1 - \lambda_i)$'s are fuzzy closed sets in (X,T). Then $cl(1 - \lambda_i) = 1 - \lambda_i$ and $int(1 - \lambda_i) = 0$ implies that $intcl(1 - \lambda_i) = int(1 - \lambda_i) = 0$. That is, $(1 - \lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Therefore we have $int(\vee_{i=1}^{\infty}(1-\lambda_i)) = 0$, where $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Hence, by theorem 3.4, (X,T) is a fuzzy Baire space. □

Definition 3.5 ([15]). A fuzzy topological space (X,T) is called a *fuzzy almost resolvable space* if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets λ_i's in (X,T) are such that $int(\lambda_i) = 0$. Otherwise (X,T) is called a *fuzzy almost irresolvable space.*

Proposition 3.13. *If the fuzzy topological space (X,T) is a fuzzy almost resolvable space, then (X,T) is a fuzzy σ-second category space.*

Proof. Let (λ_i)'s $(i = 1\ to\ \infty)$ be the fuzzy dense and fuzzy G_δ-sets in (X,T). Now $cl(\lambda_i) = 1$ implies that $1 - cl(\lambda_i) = 0$. That is, $int(1-\lambda_i) = 0$. Since (X,T) is a fuzzy almost irresolvable space, $\vee_{i=1}^{\infty}(1-\lambda_i) \neq 1$, where the fuzzy sets (λ_i)'s in (X,T) are such that $int(1-\lambda_i) = 0$. Now $\vee_{i=1}^{\infty}(1-\lambda_i) \neq 1$ implies that $1 - \vee_{i=1}^{\infty}(1-\lambda_i) \neq 0$. Hence we have $\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$, where the fuzzy sets λ_i's are fuzzy dense and fuzzy G_δ-sets in a fuzzy topological space (X,T). Thus, by proposition 3.5, (X,T) is a fuzzy σ-second category space. □

Definition 3.6 ([5]). A fuzzy topological space (X,T) is called a *fuzzy hyperconnected space* if every fuzzy open set λ is fuzzy dense in (X,T). That is, $cl(\lambda) = 1$ for all $0 \neq \lambda \in T$.

Theorem 3.5 ([13]). *If $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where λ_i's are fuzzy dense and fuzzy G_δ-sets in (X,T), then (X,T) is a fuzzy σ-Baire space.*

Proposition 3.14. *If $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where the fuzzy sets (λ_i)'s are fuzzy G_δ-sets in a fuzzy hyperconnected and fuzzy P-space (X,T), then (X,T) is a fuzzy σ-Baire space.*

Proof. Let (λ_i)'s $(i = 1\ to\ \infty)$ be the fuzzy G_δ-sets in (X,T) such that $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$. Since (X,T) is a fuzzy P-space, the fuzzy G_δ-sets (λ_i)'s in (X,T), are fuzzy open in (X,T). Also since (X,T) is a fuzzy hyperconnected space, the fuzzy open sets λ_i's in (X,T), are fuzzy dense sets in (X,T). Hence the fuzzy sets λ_i's $(i = 1\ to\ \infty)$ are fuzzy dense and fuzzy G_δ-sets in (X,T) and $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$. Hence, by theorem 3.5, (X,T) is a fuzzy σ-Baire space. □

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A Study of Fuzzy σ-Baire, Supra-Baire, and Soft Baire Spaces

Classical topology, rooted in crisp point-set concepts, has for decades supplied the language for continuity, convergence, and dimension. Over the past sixty years, however, the need to model imprecision has driven mathematicians to blend traditional topological ideas with revolutionary frameworks.

This journey began with Lotfi A. Zadeh's seminal introduction of Fuzzy Set Theory, which allowed for degrees of membership rather than binary belonging. My own extension of this logic into Neutrosophic Sets—which accounts for degrees of truth, indeterminacy, and falsity—further expanded this landscape, emphasizing that the "neutral" or "indeterminate" zone is often as critical as the membership itself. Subsequently, the introduction of Soft Set Theory by Molodtsov provided a parameterised tool for dealing with uncertainty. The resulting fuzzy topologies, supra-topologies, and soft topologies retain the elegance of their classical ancestors while admitting these nuanced layers of membership and parameterised indeterminacy.

Within this broad landscape, Baire spaces occupy a privileged position. Their defining property—that a countable union of nowhere-dense sets cannot exhaust the whole space—underpins many deep results in analysis. Extending this notion to fuzzy and soft contexts is far from trivial; the interplay between interior, closure, and degree of membership demands new techniques and fresh insight.

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